新版線形代数　改訂版　解答

２章　行列と連立１次方程式

２節　連立１次方程式と行列

P.94

練習１

⑴

⑴ $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 3 & -3 & 7 & 10 \\ -1 & 4 & -9 & -4 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -9 & -2 & -20 \\ 0 & 6 & -6 & 6 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & 9 & 2 & 20 \\ 0 & 1 & -1 & 1 \end{array}\right)$

$\to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & 0 & 11 & 11 \\ 0 & 1 & -1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 0 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right)$

$\therefore \begin{cases} x = 3 \\ y = 2 \\ z = 1 \end{cases}$

⑵ $\left(\begin{array}{ccc|c} 2 & 1 & -2 & -4 \\ 3 & -1 & 3 & 1 \\ -1 & 1 & 4 & 11 \end{array}\right) \to \left(\begin{array}{ccc|c} 0 & 3 & 6 & 18 \\ 0 & 2 & 15 & 34 \\ -1 & 1 & 4 & 11 \end{array}\right) \to \left(\begin{array}{ccc|c} 0 & 1 & 2 & 6 \\ 0 & 2 & 15 & 34 \\ 1 & -1 & -4 & -11 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & -4 & -11 \\ 0 & 1 & 2 & 6 \\ 0 & 2 & 15 & 34 \end{array}\right)$

$\to \left(\begin{array}{ccc|c} 1 & -1 & -4 & -11 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 11 & 22 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & -4 & -4 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right)$

$\therefore \begin{cases} x = -1 \\ y = 2 \\ z = 2 \end{cases}$

P94

練習 2

(1) $\left(\begin{array}{cc|c} 3 & -3 & 10 \\ -1 & 4 & -4 \end{array}\right) \to \left(\begin{array}{cc|c} 0 & 9 & -2 \\ -1 & 4 & -4 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -4 & 4 \\ 0 & 9 & -2 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -4 & 4 \\ 0 & 1 & -\frac{2}{9} \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 0 & \frac{28}{9} \\ 0 & 1 & -\frac{2}{9} \end{array}\right)$

$\therefore \quad x = \dfrac{28}{9}, \quad y = -\dfrac{2}{9}$

(2) $\left(\begin{array}{cccc|c} 1 & 1 & 2 & -1 & 3 \\ 2 & 3 & -2 & 3 & 3 \\ 1 & 1 & 3 & 3 & -6 \\ 1 & -1 & 1 & 2 & -4 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 1 & 2 & -1 & 3 \\ 0 & 1 & -6 & 5 & -3 \\ 0 & 0 & 1 & 4 & -9 \\ 0 & -2 & -1 & 3 & -7 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & 8 & -6 & 6 \\ 0 & 1 & -6 & 5 & -3 \\ 0 & 0 & 1 & 4 & -9 \\ 0 & 0 & -13 & 13 & -13 \end{array}\right)$

$\to \left(\begin{array}{cccc|c} 1 & 0 & 8 & -6 & 6 \\ 0 & 1 & -6 & 5 & -3 \\ 0 & 0 & 1 & 4 & -9 \\ 0 & 0 & -1 & 1 & -1 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & 8 & -6 & 6 \\ 0 & 1 & -6 & 5 & -3 \\ 0 & 0 & 1 & 4 & -9 \\ 0 & 0 & 0 & 5 & -10 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & 8 & -6 & 6 \\ 0 & 1 & -6 & 5 & -3 \\ 0 & 0 & 1 & 4 & -9 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right)$

$\to \left(\begin{array}{cccc|c} 1 & 0 & 8 & 0 & -6 \\ 0 & 1 & -6 & 0 & 7 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right)$

$\therefore \quad \begin{cases} x = 2 \\ y = 1 \\ z = -1 \\ w = -2 \end{cases}$

P.96

練習 3

⑴ $\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 2 & 3 & 1 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\therefore \begin{cases} x+y=1 & \cdots ① \\ y+z=0 & \cdots ② \end{cases}$ ここで $z=t$ とおくと，②より，$y=-t$

さらに $y=-t$ を①に代入すると，$x=t+1$

$\therefore \begin{cases} x=t+1 \\ y=-t \\ z=t \end{cases}$ （t は任意の実数）

⑵ $\left(\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ 1 & 1 & 2 & 2 \\ 2 & -3 & 4 & -6 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & 0 & -2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -1 & 2 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\therefore \begin{cases} x+2z=0 & \cdots ① \\ y=2 & \cdots ② \end{cases}$ ここで $z=t$ とおくと，②より，$x=-2t$

$\therefore \begin{cases} x=-2t \\ y=2 \\ z=t \end{cases}$ （t は任意の実数）

P.98

練習 4

⑴ $\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & 7 & -2 \\ 0 & 1 & -3 & 1 \end{array}\right)$

よって，逆行列は $\begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}$

⑵ $\left(\begin{array}{cc|cc} 3 & -8 & 1 & 0 \\ 2 & -5 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & -3 & 1 & -1 \\ 2 & -5 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & -3 & 1 & -1 \\ 0 & 1 & -2 & 3 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & -5 & 8 \\ 0 & 1 & -2 & 3 \end{array}\right)$

よって，逆行列は $\begin{pmatrix} -5 & 8 \\ -2 & 3 \end{pmatrix}$

⑶ $\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & -2 & -3 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{3}{2} & -\frac{1}{2} \end{array}\right)$

よって，逆行列は $\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$

P.98

練習 5

(2) $\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & -2 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 2 & -1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 2 & -1 \end{array}\right)$

$\to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & -2 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 2 & -1 \end{array}\right)$

よって，逆行列は $\begin{pmatrix} 1 & 2 & -2 \\ 0 & -1 & 1 \\ 0 & 2 & -1 \end{pmatrix}$

(2) $\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -2 & -1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4 & 1 & 2 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right)$

$\to \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -4 & 1 & 2 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 3 & -1 & -1 \\ 0 & 0 & 1 & -4 & 1 & 2 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1 & 1 \\ 0 & 1 & 0 & 3 & -1 & -1 \\ 0 & 0 & 1 & -4 & 1 & 2 \end{array}\right)$

よって，逆行列は $\begin{pmatrix} -2 & 1 & 1 \\ 3 & -1 & -1 \\ -4 & 1 & 2 \end{pmatrix}$

(3) $\left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 5 & 4 & 0 & 1 & 0 \\ 3 & 6 & 8 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -1 & 2 & -2 & 1 & 0 \\ 0 & -3 & 5 & -3 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 2 & -1 & 0 \\ 0 & 3 & -5 & 3 & 0 & -1 \end{array}\right)$

$\to \left(\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & 2 & -1 & 0 \\ 0 & 0 & 1 & -3 & 3 & -1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & 4 & -3 & 1 \\ 0 & 1 & 0 & -4 & 5 & -2 \\ 0 & 0 & 1 & -3 & 3 & -1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 16 & -18 & 7 \\ 0 & 1 & 0 & -4 & 5 & -2 \\ 0 & 0 & 1 & -3 & 3 & -1 \end{array}\right)$

よって，逆行列は $\begin{pmatrix} 16 & -18 & 7 \\ -4 & 5 & -2 \\ -3 & 3 & -1 \end{pmatrix}$

P.98

練習 6

(1) $\left(\begin{array}{cccc|cccc} 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cccc|cccc} 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$

$\to \left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 1 & 1 & 2 & -2 & 0 \\ 0 & 1 & 0 & 2 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -2 & -1 & -1 & -1 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 2 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & -3 & 2 & 1 \end{array}\right)$

$\to \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 2 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 3 & -2 & -1 \end{array}\right) \to \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & 3 & -3 & -1 \\ 0 & 1 & 0 & 0 & -2 & -4 & 3 & 2 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 3 & -2 & -1 \end{array}\right)$

よって，逆行列は $\begin{pmatrix} 2 & 3 & -3 & -1 \\ -2 & -4 & 3 & 2 \\ 0 & -1 & 1 & 0 \\ 1 & 3 & -2 & -1 \end{pmatrix}$

(2) $\left(\begin{array}{cccc|cccc} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 1 & -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & -1 & 1 & 0 & 0 & 1 & 0 \end{array}\right)$

$\to \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & 1 & 0 \end{array}\right) \to \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 & 1 & 1 \end{array}\right)$

$\to \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{array}\right) \to \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{array}\right)$

よって，逆行列は $\begin{pmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

P.99

練習 7

⑴ $\begin{pmatrix}1 & 2\\ 2 & 3\end{pmatrix}\to\begin{pmatrix}1 & 2\\ 0 & -1\end{pmatrix}\to\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}\to\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$　よって，階級は 2

⑵ $\begin{pmatrix}1 & 3 & 1\\ 2 & 4 & 1\\ 1 & 1 & 0\end{pmatrix}\to\begin{pmatrix}1 & 3 & 1\\ 0 & -2 & -1\\ 0 & -2 & -1\end{pmatrix}\to\begin{pmatrix}1 & 3 & 1\\ 0 & 2 & 1\\ 0 & 0 & 0\end{pmatrix}\to\begin{pmatrix}1 & 1 & 0\\ 0 & 2 & 1\\ 0 & 0 & 0\end{pmatrix}$

$\to\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 2\\ 0 & 0 & 0\end{pmatrix}$ よって階級は 2

⑶ $\begin{pmatrix}2 & 5 & 3 & 0\\ 1 & 3 & 1 & 1\\ 2 & 7 & 1 & 4\end{pmatrix}\to\begin{pmatrix}0 & -1 & 1 & -2\\ 1 & 3 & 1 & 1\\ 0 & 1 & -1 & 2\end{pmatrix}\to\begin{pmatrix}0 & 0 & 0 & 0\\ 1 & 3 & 1 & 1\\ 0 & 1 & -1 & 2\end{pmatrix}\to\begin{pmatrix}1 & 3 & 1 & 1\\ 0 & 1 & -1 & 2\\ 0 & 0 & 0 & 0\end{pmatrix}$

$\to\begin{pmatrix}1 & 0 & 4 & -5\\ 0 & 1 & -1 & 2\\ 0 & 0 & 0 & 0\end{pmatrix}$ よって，階級は 2

P.100

練習 8

$A=\begin{pmatrix}1 & 1 & 2\\ 2 & 1 & 5\\ 1 & 2 & 1\end{pmatrix}\to\begin{pmatrix}1 & 1 & 2\\ 0 & -1 & 1\\ 0 & 1 & -1\end{pmatrix}\to\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 1\\ 0 & 1 & -1\end{pmatrix}\to\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 1\\ 0 & 0 & 0\end{pmatrix}\to\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & -1\\ 0 & 0 & 0\end{pmatrix}$

よって　$\operatorname{rank}A=2$

$A'=\begin{pmatrix}1 & 1 & 2 & 1\\ 2 & 1 & 5 & 1\\ 1 & 2 & 1 & 1\end{pmatrix}\to\begin{pmatrix}1 & 1 & 2 & 1\\ 0 & -1 & 1 & -1\\ 0 & 1 & -1 & 0\end{pmatrix}\to\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 1 & -1\\ 0 & 1 & -1 & 0\end{pmatrix}\to\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & -1 & 1 & -1\\ 0 & 0 & 0 & -1\end{pmatrix}$

$\to\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & -1 & -1\\ 0 & 0 & 0 & -1\end{pmatrix}\to\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}\to\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{pmatrix}$

より　$\operatorname{rank}A'=3$

$\operatorname{rank}A\neq\operatorname{rank}A'$ より解なし。

節末問題

P.101

1

(1) $\left(\begin{array}{ccc|c} 3 & 1 & 1 & 6 \\ 2 & 2 & 1 & 3 \\ 2 & 1 & 3 & 6 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -2 & -3 \\ 2 & 1 & 3 & 6 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -2 & -3 \\ 0 & 1 & 7 & 6 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 9 & 9 \end{array}\right)$

$\rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right) \quad \therefore \begin{cases} x = 2 \\ y = -1 \\ z = 1 \end{cases}$

(2) $\left(\begin{array}{ccc|c} 1 & 3 & 2 & -3 \\ -4 & 2 & -6 & 3 \\ -2 & 4 & -4 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 3 & 2 & -3 \\ 0 & 14 & 2 & -9 \\ 0 & 10 & 0 & -5 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 3 & 2 & -3 \\ 0 & 14 & 2 & -9 \\ 0 & 2 & 0 & -1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 3 & 2 & -3 \\ 0 & 0 & 2 & -2 \\ 0 & 2 & 0 & -1 \end{array}\right)$

$\rightarrow \left(\begin{array}{ccc|c} 1 & 3 & 2 & -3 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 3 & 0 & -1 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -1 \end{array}\right) \quad \therefore \begin{cases} x = \frac{1}{2} \\ y = -\frac{1}{2} \\ z = -1 \end{cases}$

節末問題　つづき

P.101

2

(1) $\left(\begin{array}{ccc|c} 3 & 1 & 2 & 4 \\ 3 & 2 & 1 & 5 \\ 2 & 1 & 1 & 3 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 2 & 1 & 1 & 3 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$\therefore \begin{cases} x+z=1 & \cdots① \\ y-z=1 & \cdots② \end{cases}$　　ここで $z=t$ とおくと，①より，$x=-t+1$

さらに$z=t$ を②に代入すると，$y=t+1$

$\therefore \begin{cases} x=-t+1 \\ y=t+1 \\ z=t \end{cases}$　　（t は任意の実数）

(2) $\left(\begin{array}{ccc|c} 5 & 4 & 6 & 13 \\ 2 & 5 & -1 & 12 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -6 & 8 & -11 \\ 2 & 5 & -1 & 12 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -6 & 8 & -11 \\ 0 & 17 & -17 & 34 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -6 & 8 & -11 \\ 0 & 1 & -1 & 2 \end{array}\right)$

$\therefore \begin{cases} x-6y+8z=-11 & \cdots① \\ y-z=2 & \cdots② \end{cases}$　　ここで $z=t$ とおくと，②より，$y=t+2$

さらに $z=t$，$y=t+2$ を①に代入すると，$x=-2t+1$

$\therefore \begin{cases} x=-2t+1 \\ y=t+2 \\ z=t \end{cases}$　　（t は任意の実数）

節末問題　つづき

P.101

3

⑴ $\begin{pmatrix} 4 & 1 & 6 \\ 3 & 3 & 9 \\ 1 & 2 & 5 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & -3 \\ 1 & 1 & 3 \\ 1 & 2 & 5 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & -3 \\ 0 & 3 & 6 \\ 0 & 4 & 8 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & -3 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

$\to \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$

よって，階数は 2

⑵ $\begin{pmatrix} 3 & 7 & 6 & 2 \\ 2 & 5 & 5 & 1 \\ 3 & 6 & 4 & 5 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 & 1 \\ 2 & 5 & 5 & 1 \\ 0 & -1 & -2 & 3 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 1 & 2 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & -1 & -2 \end{pmatrix}$

$\to \begin{pmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 13 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

よって，階数は 3

⑶ $\begin{pmatrix} 2 & 4 & 3 & 3 \\ 3 & 5 & 5 & 4 \\ 1 & 3 & 1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 3 & 1 & 2 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & -1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$\to \begin{pmatrix} 1 & 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix}$

よって，階数は 2

節末問題　つづき

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(1) $\left(\begin{array}{ccc|ccc} -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 & -1 & 0 \end{array}\right)$

$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 & -1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right)$

よって，逆行列は $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(2) $\left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 2 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right)$

$\rightarrow \left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & -2 & 3 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & -1 & 2 & -3 \\ 0 & 1 & 0 & 2 & -2 & 3 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right)$

$\rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & 1 & -\frac{3}{2} \\ 0 & 1 & 0 & 2 & -2 & 3 \\ 0 & 0 & 1 & -1 & 1 & -1 \end{array}\right)$

よって，逆行列は $\begin{pmatrix} -\frac{1}{2} & 1 & -\frac{3}{2} \\ 2 & -2 & 3 \\ -1 & 1 & -1 \end{pmatrix}$

(3) $\left(\begin{array}{cccc|cccc} 1 & -1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{array}\right) \rightarrow \left(\begin{array}{cccc|cccc} 1 & -1 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{array}\right)$

$\rightarrow \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{array}\right)$

よって，逆行列は $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

節末問題　つづき

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行と列の基本変形により，Aを$\begin{pmatrix} E_r & * \\ O & O \end{pmatrix}$ の形に変形すると，$r \leqq m$，$r \leqq n$

∴　$\mathrm{rank}\, A \leqq m$，$\mathrm{rank}\, A \leqq n$

6　与えられた連立方程式を掃き出し法で解くと

$$\left(\begin{array}{ccc|c} -1 & -1 & 1 & -3 \\ 2 & 3 & -1 & 8 \\ 3 & 4 & -2 & 11 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

∴ $\begin{cases} x+y-z=3 & \cdots① \\ y+z=2 & \cdots② \end{cases}$　　ここで $z=t$ とおくと，②より，$y=-t+2$

さらに $z=t$，$y=-t+2$ を ① に代入すると，$x=2t+1$

∴ $\begin{cases} x=2t+1 \\ y=-t+2 \\ z=t \end{cases}$　　（tは任意の実数）

∴ $\vec{v}=\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 2t+1 \\ -t+2 \\ t \end{pmatrix}=\begin{pmatrix} 2t \\ -t \\ t \end{pmatrix}+\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}=t\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}+\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$