２章　微分法

２節　導関数

A 問題

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⑴　$f'(-1)=\lim_{h\to 0}\dfrac{\left\{(-1+h)^2-(-1+h)\right\}-\left\{(-1)^2-(-1)\right\}}{h}=\lim_{h\to 0}\dfrac{-3h+h^2}{h}=\lim_{h\to 0}(-3+h)=-3$

⑵　$f'(-1)=\lim_{h\to 0}\dfrac{\left\{2(-1+h)^2-1\right\}-\left\{2(-1)^2-1\right\}}{h}=\lim_{h\to 0}\dfrac{-4h+2h^2}{h}=\lim_{h\to 0}(-4+2h)=-4$

⑶　$f'(-1)=\lim_{h\to 0}\dfrac{\left\{(-1+h)^3-1\right\}-\left\{(-1)^3-1\right\}}{h}=\lim_{h\to 0}\dfrac{3h-3h^2+h^3}{h}=\lim_{h\to 0}(3-3h+h^2)=3$

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$\lim_{h\to +0}\dfrac{f(1+h)-f(1)}{h}=\lim_{h\to +0}\dfrac{2(1+h)-1-(2\cdot 1-1)}{h}=\lim_{h\to +0}\dfrac{2h}{h}=0$

$\lim_{h\to -0}\dfrac{f(1+h)-f(1)}{h}=\lim_{h\to -0}\dfrac{(1+h)^2-1^2}{h}=\lim_{h\to -0}\dfrac{\cancel{h}(2+h)}{\cancel{h}}=2$

よって，$f'(1)=2$ となり存在するから微分可能である。

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⑴　$f'(x)=\lim_{h\to 0}\dfrac{\left\{2(x+h)+1\right\}-\left\{2x+1\right\}}{h}=\lim_{h\to 0}\dfrac{2h}{h}=\lim_{h\to 0}2=2$

⑵　$f'(x)=\lim_{h\to 0}\dfrac{\left\{(x+h)^2+(x+h)\right\}-\left\{x^2+x\right\}}{h}=\lim_{h\to 0}\dfrac{2xh+h^2+h}{h}=\lim_{h\to 0}(2x+h+1)=2x+1$

⑶　$f'(x)=\lim_{h\to 0}\dfrac{\left\{(x+h)^3-(x+h)\right\}-\left\{x^3-x\right\}}{h}=\lim_{h\to 0}\dfrac{3x^2h+3xh^2+h^3-h}{h}=\lim_{h\to 0}(3x^2+3xh+h^2-1)=3x^2-1$

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⑴　$y'=6x-5$

⑵　$y'=-3x^2+4x+1$

⑶　$y'=-6x^2-2x+5$

⑷　$y'=\dfrac{1}{2}x^2-\dfrac{1}{2}x-1$

⑸　$y'=-2x^2+5$

⑹　$y'=4x^3-10x+\dfrac{2}{3}$

92　$f'(x)=3x^2-2x+1$

$f'(1)=3-2+1=2$，　$f'(0)=1$，　$f'(-2)=12+4+1=17$

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(1) $y' = 1 \cdot (2x+6) + (x-1) \cdot 2 = 4x+4$

(2) $y' = 3 \cdot (x^2 - 2x - 1) + (3x+2)(2x-2) = 9x^2 - 8x - 7$

(3) $y' = 2x(x^3 - 2) + (x^2+3) \cdot 3x^2 = 5x^4 + 9x^2 - 4x$

(4) $y' = 1 \cdot (2x+1)(3x-1) + (x+1) \cdot 2 \cdot (3x-1) + (x+1)(2x+1) \cdot 3 = 18x^2 + 14x$

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(1) $y' = -\dfrac{4}{(4x-1)^2}$

(2) $y' = \dfrac{2(x+1) - (2x-1) \cdot 1}{(x+1)^2} = \dfrac{3}{(x+1)^2}$

(3) $y' = \dfrac{1 \cdot (x^2+2) - (x+1) \cdot 2x}{(x^2+2)^2} = -\dfrac{x^2+2x-2}{(x^2+2)^2}$

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(1) $y' = (x^{-4})' = -4x^{-5} = -\dfrac{4}{x^2}$

(2) $y' = (2x^{-2})' = -4x^{-3} = -\dfrac{4}{x^3}$

(3) $y' = \left(-\dfrac{1}{2}x^{-6}\right)' = 3x^{-7} = \dfrac{3}{x^7}$

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⑴ $u=5x+4$ とおくと $y=u^2$ であり， $\dfrac{dy}{du}=2u$ ， $\dfrac{du}{dx}=5$ だから $y'=2u\cdot 5=10(5x+4)$

⑵ $u=4x-1$ とおくと $y=u^3$ であり， $\dfrac{dy}{du}=3u^2$ ， $\dfrac{du}{dx}=4$ だから $y'=3u^2\cdot 4=12(4x-1)^2$

⑶ $u=2x^2+1$ とおくと $y=u^4$ であり， $\dfrac{dy}{du}=4u^3$ ， $\dfrac{du}{dx}=4x$ だから $y'=4u^3\cdot 4x=16x(2x^2+1)^3$

⑷ $u=3x^2-x+1$ とおくと $y=u^3$ であり， $\dfrac{dy}{du}=3u^2$ ， $\dfrac{du}{dx}=6x-1$ だから

$y'=3u^2\cdot(6x-1)=3(6x-1)(3x^2-x+1)^2$

⑸ $u=x-1$ とおくと $y=u^{-2}$ であり， $\dfrac{dy}{du}=-2u^{-3}$ ， $\dfrac{du}{dx}=1$ だから

$y'=-2u^{-3}\cdot 1=-\dfrac{2}{(x-1)^3}$

⑹ $u=x^2+3$ とおくと $y=u^{-4}$ であり， $\dfrac{dy}{du}=-4u^{-5}$ ， $\dfrac{du}{dx}=2x$ だから

$y'=-4u^{-5}\cdot 2x=-\dfrac{8x}{(x^2+3)^5}$

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⑴ $y'=\left(x^{\frac{3}{2}}\right)'=\dfrac{3}{2}u^{\frac{1}{2}}=\dfrac{3}{2}\sqrt{x}$

⑵ $y'=\left\{(x^2+1)^{\frac{1}{2}}\right\}'=\dfrac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x=\dfrac{x}{\sqrt{x^2+1}}$

⑶ $y'=\left\{(3x^2+1)^{\frac{1}{3}}\right\}'=\dfrac{1}{3}(3x^2+1)^{-\frac{2}{3}}\cdot 6x=\dfrac{2x}{\sqrt[3]{(3x^2+1)^2}}$

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⑴ 両辺を 2 乗して $y^2=x+1$ ∴ $x=y^2-1$ ∴ $\dfrac{dx}{dy}=2y$ ∴ $y'=\dfrac{1}{2y}=\dfrac{1}{2\sqrt{x+1}}$

⑵ 両辺を 3 乗して $y^3=\dfrac{27}{x}$ ∴ $x=\dfrac{27}{y^3}$ ∴ $\dfrac{dx}{dy}=-\dfrac{81}{y^4}$

$y'=\dfrac{1}{-\dfrac{81}{y^4}}=-\dfrac{1}{81}y^4=-\dfrac{1}{81}\cdot\dfrac{81}{\sqrt[3]{x^4}}=-\dfrac{1}{\sqrt[3]{x^4}}$

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(1) $y' = -\sin 2x \cdot 2 = -2\sin 2x$

(2) $y' = \cos(1-x)\cdot(-1) = -\cos(1-x)$

(3) $y' = \dfrac{1}{\cos^2 3x}\cdot 3 = \dfrac{3}{\cos^2 3x}$

(4) $y' = 2\sin x \cdot \cos x = 2\sin x \cos x$

(5) $y' = 3\cos^2 x \cdot(-\sin x) = -3\cos^2 x \sin x$

(6) $y' = 2\tan x \cdot \dfrac{1}{\cos^2 x} = \dfrac{2\tan x}{\cos^2 x}$

(7) $y' = -\dfrac{\cos x}{\sin^2 x}$

(8) $y' = -\dfrac{-\sin^2 x}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x}$

(9) $y' = \dfrac{-\sin x \cdot x - \cos x \cdot 1}{x^2} = -\dfrac{x\sin x + \cos x}{x^2}$

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(1) $y' = \dfrac{1}{\sqrt{1-(2x)^2}}\cdot 2 = \dfrac{2}{\sqrt{1-4x^2}}$

(2) $y' = \dfrac{1}{\sqrt{1-(3x)^2}}\cdot 3 = -\dfrac{3}{\sqrt{1-9x^2}}$

(3) $y' = \dfrac{1}{1+(2x)^2}\cdot 2 = \dfrac{2}{1+4x^2}$

(4) $y' = \dfrac{1}{\sqrt{1-\left(\dfrac{x}{3}\right)^2}}\cdot\dfrac{1}{3} = \dfrac{1}{\sqrt{9-x^2}}$

(5) $y' = \dfrac{1}{1+\left(\dfrac{x}{2}\right)^2}\cdot\dfrac{1}{2} = \dfrac{2}{4+x^2}$

(6) $y' = \dfrac{1}{1+x}\cdot\dfrac{1}{2\sqrt{x}} = \dfrac{1}{2(x+1)\sqrt{2}}$

101

(1) $y' = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

(2) $y' = \frac{1}{3x+1} \cdot 3 = \frac{3}{3x+1}$

(3) $y' = \frac{1}{x \log 3}$

(4) $y' = 3x^2 \log x + x^3 \cdot \frac{1}{x} = x^2 \left(3\log x + 1 \right)$

(5) $y' = 1 \cdot \log_2 x + x \cdot \frac{1}{x \log 2} = \frac{\log x}{\log 2} + \frac{1}{\log 2} = \frac{1}{\log 2}\left(\log x + 1 \right)$

(6) $y' = \frac{\frac{1}{x} \cdot x - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x^2}$

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(1) $y' = \frac{1}{2x-1} \cdot 2 = \frac{2}{2x-1}$

(2) $y' = \frac{1}{x^2 - x} \cdot \left(2x-1 \right) = \frac{2x-1}{x^2-x}$

(3) $y' = \frac{1}{\cos x} \cdot \left(-\sin x \right) = -\tan x$

103　両辺の対数をとると　$\log y = \log \frac{\left(x-2 \right)^3}{\left(x-1 \right)^2} = \log\left(x-2 \right)^3 - \log\left(x-1 \right)^2 = 3\log\left(x-2 \right) - 2\log\left(x-1 \right)$

この両辺を x で微分すると　$\frac{y'}{y} = \frac{3}{x-2} - \frac{2}{x-1} = \frac{x+1}{\left(x-2 \right)\left(x-1 \right)}$

$\therefore \quad y' = \frac{x+1}{\left(x-2 \right)\left(x-1 \right)} \cdot \frac{\left(x-2 \right)^3}{\left(x-1 \right)^2} = \frac{\left(x+1 \right)\left(x-2 \right)^2}{\left(x-1 \right)^3}$

104

(1) $y' = e^{3x+1} \cdot 3 = 3e^{3x+1}$

(2) $y' = 1 \cdot e^x + xe^x = \left(x+1 \right)e^x$

(3) $y' = 2^{1-x} \log 2 \cdot \left(-1 \right) = -2^{1-x} \log 2$

105

(1) $y' = e^x \cos x + e^x \left(-\sin x \right) = e^x \left(\cos x - \sin x \right)$

(2) $y' = \frac{e^x \left(x+1 \right) - e^x \cdot 1}{\left(x+1 \right)^2} = \frac{xe^x}{\left(x+1 \right)^2}$

(3) $y' = e^{-x^2} \cdot \left(-2x \right) = -2xe^{-x^2}$

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(1) $y' = 6x^2 - 6x + 4 \quad y'' = 12x - 6$

(2) $y' = \dfrac{1}{1+x^2} \quad y'' = -\dfrac{2x}{\left(1+x^2\right)^2}$

(3) $y' = \sin x + x\cos x \quad y'' = \cos x + \cos x + x\left(\sin x\right) = 2\cos x - x\sin x$

107

(1) $y''' = \left(5x^4 + 8x^3 - 9x^2\right)'' = \left(20x^3 + 24x^2 - 18x\right)' = 60x^2 + 48x - 18$

(2) $y''' = \left(2\cos 2x\right)'' = \left(-4\sin 2x\right)' = -8\cos 2x$

(3) $y''' = \left(\dfrac{3}{2}\sqrt{x}\right)'' = \left(\dfrac{3}{4\sqrt{x}}\right)' = -\dfrac{3}{8\sqrt{x^3}}$

108

(1) $y' = -e^{-x}, \quad y'' = e^{-x}, \quad y''' = -e^{-x}, \quad y^{(4)} = e^{-x} \quad \cdots \therefore \; y^{(n)} = \left(-1\right)^n e^{-x}$

(2) $y' = e^{2x} + x\cdot 2e^{2x} = \left(2x+1\right)e^{2x}, \quad y'' = 2e^{2x} + \left(2x+1\right)\cdot 2e^{2x} = 2\left(2x+2\right)e^{2x},$

$y''' = 2^2 e^{2x} + 2\left(2x+2\right)\cdot 2e^{2x} = 2^2\left(2x+3\right)e^{2x}, \quad \cdots \quad y^{(4)} = \quad \therefore \quad y^{(n)} = 2^{n-1}\left(2x+n\right)e^{2x}$

(3) $y' = -\dfrac{1}{\left(x-1\right)^2}, \quad y'' = \dfrac{2\cdot 1}{\left(x-1\right)^3}, \quad y''' = -\dfrac{3\cdot 2\cdot 1}{\left(x-1\right)^4}, \quad y^{(4)} \dfrac{4\cdot 3\cdot 2\cdot 1}{\left(x-1\right)^5} \quad \cdots$

$\therefore \quad y^{(n)} = \dfrac{\left(-1\right)^n n!}{\left(x-1\right)^{n+1}}$

109 $y' = e^{-x} + x\cdot\left(-e^{-x}\right) = \left(1-x\right)e^{-x}, \quad y'' = -1\cdot e^{-x} + \left(1-x\right)\cdot\left(-e^{-x}\right) = \left(x-2\right)e^{-x}$

$\therefore \quad y'' + 2y' + y = \left(x-2\right)e^{-x} + 2\cdot\left(1-x\right)e^{-x} + xe^{-x} = 0$

B 問題

110

(1) $y' = \lim_{h\to 0}\dfrac{\dfrac{1}{x+h}-\dfrac{1}{x}}{h} = \lim_{h\to 0}\dfrac{\cancel{x}-\left(\cancel{x}+\cancel{h}\right)}{\cancel{h}\cdot\left(x+h\right)x} = \lim_{h\to 0}\dfrac{-1}{\left(x+h\right)x} = -\dfrac{1}{x^2}$

(2) $y' = \lim_{h\to 0}\dfrac{\cos\left(x+h\right)-\cos x}{h} = \lim_{h\to 0}\dfrac{\cos x\cos h-\sin x\sin h-\cos x}{h}$

$= \lim_{h\to 0}\left(\dfrac{-\cos x\left(1-\cos h\right)}{h}-\dfrac{\sin x\sin h}{h}\right) = \lim_{h\to 0}\left(\dfrac{-\cos x\left(1-\cos h\right)\left(1+\cos h\right)}{h\left(1+\cos h\right)}-\dfrac{\sin h}{h}\cdot\sin x\right)$

$= \lim_{h\to 0}\left(-\dfrac{\cos x\sin h}{h\left(1+\cos h\right)}-\dfrac{\sin h}{h}\cdot\sin x\right) = \lim_{h\to 0}\left(-\dfrac{h\sin^2 h\cos x}{h^2\left(1+\cos h\right)}-\dfrac{\sin h}{h}\cdot\sin x\right)$

$= -\sin x$

111

(1) $x^2+y^2=1$

$2x+2y\dfrac{dy}{dx}=0$

$\therefore\ \dfrac{dy}{dx}=-\dfrac{x}{y}$

(2) $x^2+xy+2y^2=1$

$2x+1\cdot y+x\dfrac{dy}{dx}+4y\dfrac{dy}{dx}=0$

$\left(x+4y\right)\dfrac{dy}{dx}=-2x-y$

$\therefore\ \dfrac{dy}{dx}=-\dfrac{2x+y}{x+4y}$

(3) $x^{\frac{1}{3}}+y^{\frac{1}{3}}=1$

$\dfrac{1}{3}x^{-\frac{2}{3}}+\dfrac{1}{3}y^{-\frac{2}{3}}\cdot\dfrac{dy}{dx}=0$

$\therefore\ \dfrac{dy}{dx}=-\dfrac{x^{-\frac{2}{3}}}{y^{-\frac{2}{3}}}=-\left(\dfrac{y}{x}\right)^{\frac{2}{3}}$

112

(1) $y'=3x^2\left(x^2+1\right)^3+x^3\cdot 3\left(x^2+1\right)^2\cdot 2x=3x^2\left(x^2+1\right)^2\left(3x^2+1\right)$

(2) $y'=2\left(x^4+2x^2+3\right)\cdot\left(4x^3+4x\right)=8x\left(x^4+2x^2+3\right)\left(x^2+1\right)$

(3) $y'=\dfrac{2\left(1-x\right)-\left(2x+1\right)\left(-1\right)}{\left(1-x\right)^2}=\dfrac{3}{\left(1-x\right)^2}$

(4) $y'=\dfrac{1\cdot\sqrt{x+1}-\left(x-1\right)\cdot\dfrac{1}{2\sqrt{x+1}}}{x+1}=\dfrac{x+3}{2\left(x+1\right)\sqrt{x+1}}$

(5) $y'=\dfrac{-2x}{2\sqrt{1-x^2}}=-\dfrac{x}{\sqrt{1-x^2}}$

(6) $y'=\dfrac{3}{\cancel{4}}\left(2x^2+1\right)^{-\frac{1}{4}}\cdot\cancel{4}x=\dfrac{3x}{\sqrt[4]{2x^2+1}}$

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⑴ 両辺の対数をとると $\log y = \log \sqrt[3]{\dfrac{x-1}{x+1}} = \dfrac{1}{3}\left(\log(x-1) - \log(x+1)\right)$

この両辺を x で微分すると $\dfrac{y'}{y} = \dfrac{1}{3}\left(\dfrac{1}{x-1} - \dfrac{1}{x+1}\right) = \dfrac{2}{3(x-1)(x+1)}$

$\therefore\ y' = \dfrac{2}{3(x-1)(x+1)}\sqrt[3]{\dfrac{x-1}{x+1}}$

⑵ 両辺の対数をとると $\log y = \log x^{\frac{1}{x}} = \dfrac{1}{x}\log x$

この両辺を x で微分すると $\dfrac{y'}{y} = -\dfrac{1}{x^2}\log x + \dfrac{1}{x}\cdot\dfrac{1}{x} = \dfrac{1}{x^2}(1 - \log x)$

$\therefore\ y' = x^{\frac{1}{x}}\cdot\dfrac{1}{x^2}(1-\log x) = x^{\frac{1}{x}-2}(1-\log x)$

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⑴ $y' = \cos x\cdot\cos^2 x + \sin x\cdot 2\cos x(-\sin x) = \cos^3 x - 2\sin^2 x\cos x = 3\cos^3 x - 2\cos x$

⑵ $y' = \left\{\log\dfrac{x^2+1}{x}\right\}' = \left\{\log(x^2+1) - \log x\right\}' = \dfrac{2x}{x^2+1} - \dfrac{1}{x} = \dfrac{2x^2-(x^2+1)}{x(x^2+1)} = \dfrac{x^2-1}{x(x^2+1)}$

⑶ $y' = \dfrac{(\cos x+\sin x)^2 + (\sin x-\cos x)^2}{(\sin x-\cos x)^2} = \dfrac{\cos^2 x + \cancel{2\cos x\sin x} + \sin^2 x + \sin^2 x - \cancel{2\sin x\cos x} + \cos^2 x}{(\sin x-\cos x)^2}$

$= \dfrac{2}{(\sin x+\cos x)^2}$

⑷ $y' = \cos x\, e^{\sin x}$

⑸ $y' = 2e^{2x}\sin^2 x + e^{2x}2\sin x\cos x = 2e^{x}\sin x(\sin x + \cos x)$

⑹ $y' = \left(\log|1-\cos x| - \log|1+\cos x|\right)' = \dfrac{\sin x}{1-\cos x} - \dfrac{-\sin x}{1+\cos x}$

$= \dfrac{\sin x + \cancel{\sin x\cos x} + \sin x - \cancel{\sin x\cos x}}{1-\cos^2 x} = \dfrac{2\sin x}{\sin^2 x} = \dfrac{2}{\sin x}$

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⑴ $y' = \dfrac{1}{2}\left(\sqrt{x^2+1} + \dfrac{x^2}{\sqrt{x^2+1}} + \dfrac{1+\dfrac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}}\right)$

$= \dfrac{1}{2}\left(\sqrt{x^2+1} + \dfrac{x^2}{\sqrt{x^2+1}} + \dfrac{\cancel{\sqrt{x^2+1}+x}}{\cancel{\left(x+\sqrt{x^2+1}\right)}\sqrt{x^2+1}}\right) = \dfrac{1}{2}\left(\sqrt{x^2+1} + \dfrac{x^2+1}{\sqrt{x^2+1}}\right)$

$= \dfrac{1}{2}\left(\sqrt{x^2+1} + \sqrt{x^2+1}\right) = \sqrt{x^2+1}$

$y' = \dfrac{1}{2}\left(\sqrt{1-x^2} - \dfrac{x^2}{\sqrt{1-x^2}} + \dfrac{1}{\sqrt{1-x^2}}\right) = \dfrac{1}{2}\left(\sqrt{1-x^2} + \dfrac{1-x^2}{\sqrt{1-x^2}}\right)$

$= \dfrac{1}{2}\left(\sqrt{1-x^2} + \sqrt{1-x^2}\right) = \sqrt{1-x^2}$

⑵

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⑴ 与式を①とする。

（Ⅰ） $n=1$ のとき

$$y'=-\sin x=\cos\left(x+\frac{\pi}{2}\right)$$ だから， $n=1$ のとき①は成り立つ。

（Ⅱ） $n=k$ のとき①が成り立つと仮定すると

$n=k+1$ のとき

$$y^{(k+1)}=\left\{y^{(k)}\right\}'=\left\{\cos\left(x+\frac{k\pi}{2}\right)\right\}'=-\sin\left(x+\frac{k\pi}{2}\right)=\cos\left\{\left(x+\frac{k\pi}{2}\right)+\frac{\pi}{2}\right\}$$

$$=\cos\left(x+\frac{(k+1)\pi}{2}\right)$$ だから， $n=k+1$ のときも成り立つ。

（Ⅰ），（Ⅱ）より，任意の自然数 n について，①は成り立つ。

⑵ 与式を①とする。

（Ⅰ） $n=1$ のとき

$$y'=2\cos(2x+1)=2\sin\left(2x+1+\frac{\pi}{2}\right)$$ だから， $n=1$ のとき①は成り立つ。

（Ⅱ） $n=k$ のとき①が成り立つとすると

$n=k+1$ のとき

$$y^{(k+1)}=\left\{y^{(k)}\right\}'=\left\{2^k\sin\left(2x+1+\frac{k\pi}{2}\right)\right\}'=2^k\cdot\cos\left(2x+1+\frac{k\pi}{2}\right)$$

$$=2^{k+1}\sin\left\{\left(2x+1+\frac{k\pi}{2}\right)+\frac{\pi}{2}\right\}=2^{k+1}\sin\left(2x+1+\frac{(k+1)\pi}{2}\right)$$

だから， $n=k+1$ のときも成り立つ。

（Ⅰ），（Ⅱ）より，任意の自然数 n について，①は成り立つ。

発展問題

117

⑴　$(\text{左辺}) = \left(\sinh x\right)' = \left(\frac{e^x - e^{-x}}{2}\right)' = \frac{e^x + e^{-x}}{2} = \cosh x = (\text{右辺})$

⑵　$(\text{左辺}) = \left(\cosh x\right)' = \left(\frac{e^x + e^{-x}}{2}\right)' = \frac{e^x - e^{-x}}{2} = \sinh x = (\text{右辺})$

⑶　$(\text{左辺}) = \cosh^2 x - \sinh^{2x} = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 = \frac{\left(\cancel{e^{2x}} + 2 + \cancel{e^{-x}}\right) - \left(\cancel{e^{2x}} - 2 + \cancel{e^{-x}}\right)}{4} = 1 = (\text{右辺})$

⑷　$(\text{左辺}) = \left(\tanh x\right)' = \left(\frac{\sinh x}{\cosh x}\right)' = \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x} = (\text{右辺})$