1章　数と式　解答

1節　整式

練習 1

$6a^2x^3y$ の次数は 6，係数は 6

x に着目したときの次数は 3，係数は $6a^2y$

y に着目したときの次数は 1，係数は $6a^2x^3$

練習 2

$2x^2y+3x^3+y^2-5x^2+x-1 = y^2+2x^3y+3x^2-5x^2+x-1$,

y^2 の係数は 1，y の係数は $2x^2$

定数項は $3x^3-5x^2+x-1$

練習 3

(1)　$A = x^3-2x^2+3,\ B = 2x^2-x+1$

$A+B = x^3-x+4$,

$A-B = x^3-4x^2+x+2$

(2)　$A = -3x+5+2x^2,\ B = x-x^3-4+3x^2$

$A+B = -x^3+5x^2-2x+1$,

$A-B = x^3-x^2-4x+9$

練習 4

(1)　$x \times (2x)^2 \times (-x)^5 = x \cdot 4x^2 \cdot (-x^5) = -4x^8$

(2)　$-8a^2b \times \left(\dfrac{1}{2}ab\right)^2 = -8^2 a^2b \cdot \dfrac{a^2b^2}{4} = -2a^4b^3$

(3)　$\left(-ab^3c^2\right)^2 \times \left(-2a^2bc\right)^3 = a^2b^6c^4 \cdot \left(-8a^6b^3c^3\right) = -8a^8b^9c^7$

練習 5

(1)　$(x-1)(x^2+x+1) - x^3+x^2+x-x^2-x-1 = x^3-1$

(2)　$(x^2-3x+2)(x^2-1) = x^4-x^2-3x^3+3x+2x^2-2 = x^4-3x^3+x^2+3x-2$

(3)　$(2x^2+2xy-y^2)(3x+y) = 6x^3+2x^2y+6x^2y+2xy^2-3xy^2-y^3 = 6x^3+8x^2y-xy^2-y^3$

(4)　$(3x^3-4x+5)(x^2-x+4) = 3x^5-3x^4+12x^3-4x^3+4x^2-16x+5x^2-5x+20$

$$= 3x^5-3x^4+8x^3+9x^2-21x+20$$

練習 6

(1)　$(3x+4y)^2 = 9x^2+24xy+16y^2$

(2)　$(5x-2y)^2 = 25x^2-20xy+4y^2$

(3)　$(4x+5y)(4x-5y) = 16x^2-25y^2$

(4)　$(x+7y)(x-2y) = x^2+5xy-14y^2$

(5)　$(4x-3y)(5x+6y) = 20x^2+9xy-18y^2$

練習 7

$$\left(a + b \right)^3 = \left(a + b \right)\left(a + b \right)\left(a + b \right) = \left(a^2 + 2ab + b^2 \right)\left(a + b \right)$$
$$= a^3 + a^2 b + 2a^2 b + 2ab^2 + ab^2 + b^3 = a^3 + 3a^2 b + 3ab^2 + b^3$$
$$\left(a - b \right)^3 = \left(a - b \right)\left(a - b \right)\left(a - b \right) = \left(a^2 - 2ab + b^2 \right)\left(a - b \right)$$
$$= a^3 - a^2 b - 2a^2 b + 2ab^2 + ab^2 - b^3 = a^3 - 3a^2 b + 3ab^2 - b^3$$

練習 8

(1) $\left(x + 1 \right)^3 = x^3 + 3x^2 + 3x + 1$

(2) $\left(x - 2 \right)^3 = x^3 - 6x^2 + 12x - 8$

(3) $\left(2x + y \right)^3 = 8x^3 + 12x^2 y + 6xy^2 + y^3$

(4) $\left(3x - 2y \right)^3 = 27x^3 - 54x^2 y + 36xy^2 - 8y^3$

練習 9

(1) $\left(a - b - 1 \right)\left(a - b + 3 \right) = \left(a - b \right)^2 + 2\left(a - b \right) - 3 = a^2 - 2ab + b^2 + 2a - 2b - 3$

(2) $\left(x + 2y + z \right)\left(x - 2y + z \right) = \left(x + 2 \right)^2 - \left(2y \right)^2 = x^2 + 2xz + z^2 - 4y^2$

練習 10

(1) $\left(a - b + c \right)^2 = a^2 - \left(-b \right)^2 + c^2 + 2a\left(-b \right) + 2\left(-b \right)c + 2ca$
$$= a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$$

(2) $\left(2x + 3y + z \right)^2 = \left(2x \right)^2 + \left(3y \right)^2 + z^2 + 2 \cdot 2x \cdot 3y + 2 \cdot 3y \cdot z + 2z \cdot 2x$
$$= 4x^2 + 9y^2 + z^2 + 12xy + 6yz + 4zx$$

(3) $\left(-p + 2q - 2r \right)^2 = \left(-p \right)^2 + \left(2q \right)^2 + \left(-2r \right)^2 + 2\left(-p \right) \cdot 2q + 2 \cdot 2q\left(-2r \right) + 2\left(-2r \right) \cdot \left(-q \right)$
$$= p^2 + 4q^2 + 4r^2 - 4pq - 8qr + 4rp$$

練習 11

(1) $\left(a + b \right)\left(a^2 - ab + b^2 \right) = a\left(a^2 - ab + b^2 \right) + b\left(a^2 - ab + b^2 \right)$
$$= a^3 - a^2 b + ab^2 + a^2 b - ab^2 + b^3 = a^3 + b^3$$

(2) $\left(a - b \right)\left(a^2 + ab + b^2 \right) = a\left(a^2 + ab + b^2 \right) - b\left(a^2 + ab + b^2 \right)$
$$= a^3 + a^2 b + ab^2 - a^2 b - ab^2 - b^3 = a^3 - b^3$$

練習 12

(1) $\left(a + 2 \right)\left(a^2 - 2a + 4 \right) = \left(a + 2 \right)\left(a^2 - 2 \cdot a + 2^2 \right) = a^3 + 2^3 = a^3 + 8$

(2) $\left(3a - b \right)\left(9a^2 + 3ab + b^2 \right) = \left(3a - b \right)\left\{ \left(3a \right)^2 + 3a \cdot b + b^2 \right\}$
$$= \left(3a \right)^3 - b^3 = 27a^3 - b^3$$

練習 13

(1) $(x+1)^3(x-1)^3 = \{(x+1)(x-1)\}^3 = (x^2-1)^3 = x^6 - 3x^4 + 3x^2 - 1$

(2) $(x^2+1)(x+1)(x-1) = (x^2+1)(x^2-1) = x^4 - 1$

(3) $(2a+b-c)(2a-b+c) = \{2a+(b-c)\}\{2a-(b-c)\}$
$$= 4a^2 - (b-c)^2 = 4a^2 - b^2 + 2bc - c^2$$

(4) $(x+2y+3z)(x-2y-3z) = \{x+(2y+3z)\}\{x-(2y+3z)\}$
$$= x^2 - (2y+3z)^2 = x^2 - (4y^2+12yz+9z^2) = x^2 - 4y^2 - 12yz - 9z^2$$

練習 14

(1) $(x-1)(x-2)(x-3)(x-4) = \{(x-1)(x-4)\}\{(x-2)(x-3)\} = (x^2-5x+4)(x^2-5x+6)$
$= (x^2-5x)^2 + 10(x^2-5x) + 24 = x^4 - 10x^3 + 25x^2 + 10x^2 - 56x + 24 = x^4 - 10x^3 + 35x^2 - 50x + 24$

(2) $(x+1)(x+3)(x-2)(x-4) = \{(x+1)(x+2)\}\{(x+3)(x-4)\} = (x^2-x-2)(x^2-x-12)$
$= (x^2-x)^2 - 14(x^2-x) + 24 = x^4 - 2x^3 + x^2 - 14x^2 + 14x + 24 = x^4 - 2x^3 - 13x^2 + 14x + 24$

練習 15

(1) $x^2y - xy^2 = xy(x-y)$

(2) $a(x-y) - b(y-x) = (a+b)(x-y)$

(3) $a(b-1) - b + 1 = (a-1)(b-1)$

練習 16

(1) $16x^2 + 8xy + y^2 = (4x+y)^2$

(2) $4x^2 - 20xy + 25y^2 = (2x-5y)^2$

(3) $9x^2 - 16y^2 = (3x+4y)(3x-4y)$

(4) $3x^4y^2 - 12x^2y^4 = 3x^2y^2(x^2-4y^2) = 3x^2y^2(x+2y)(x-2y)$

(5) $x^2 + 2xy - 15y^2 = (x+5y)(x-3y)$

(6) $x^2y - 7xy^2 + 6y^3 = y(x-y)(x-6y)$

練習 17

(1) $2x^2 + 7x + 3 = (x+3)(2x+1)$

(2) $5x^2 + 2x - 3 = (x+1)(5x-3)$

(3) $6x^2 - 13x + 6 = (2x-3)(3x-2)$

(4) $2x^2 - xy - y^2 = (x-y)(2x+y)$

(5) $4a^2 - 16ab + 15b^2 = (2a-3b)(2a-5b)$

(6) $4a^2 - 5ab - 6b^2 = (a-2b)(4a+3b)$

練習 18

(1) $x^3 + 64 = (x + 4)(x^2 - 4x + 16)$

(2) $x^3 - 1 = (x - 1)(x^2 + x + 1)$

(3) $8x^3 + 125y^3 = (2x + 5y)(4x^2 - 10xy + 25y^2)$

(4) $2x^3 - 16y^3 = 2(x^3 - 8y^3) = 2(x - 2y)(x^2 + 2xy + 4y^2)$

練習 19

(1) $a^2 - (b - c)^2 = (a + b - c)(a - b + c)$

(2) $(x + y)^2 + 5(x + y) + 6 = (x + y + 2)(x + y + 3)$

(3) $(x + y - 1)(x + y + 1) - 3 = (x + y)^2 - 4 = (x + y + 2)(x + y - 2)$

(4) $(2x + y - 2)(2x + y + 5) + 12 = (2x + y)^2 + 3(2x + y) + 2 = (2x + y + 1)(2x + y + 2)$

練習 20

(1)
$$
\begin{aligned}
x^2 + xz - yz - y^2 &= (x - y)z + (x^2 - y^2) \\
&= (x - y)z + (x + y)(x - y) \\
&= (x - y)(x + y + z)
\end{aligned}
$$

(2)
$$
\begin{aligned}
x^2 - 2xy + 4y - 4 &= (-2x + 4)y + x^2 - 4 \\
&= -2(x - 2)y + (x + 2)(x - 2) \\
&= (x - 2)(x - 2y + 2)
\end{aligned}
$$

(3)
$$
\begin{aligned}
a^3 + a^2b - b - 1 &= (a^2 - 1)b + (a^3 - 1) \\
&= (a + 1)(a - 1)b + (a - 1)(a^2 + a + 1) \\
&= (a - 1)\{(a + 1)b + a^2 + a + 1\} \\
&= (a - 1)(a^2 + ab + a + b + 1)
\end{aligned}
$$

(4)
$$
\begin{aligned}
2x^2 + 6xy + x - 3y - 1 &= (6x - 3)y + 2x^2 + x - 1 \\
&= 3(2x - 1)y + (2x - 1)(x + 1) \\
&= (2x - 1)(x + 3y + 1)
\end{aligned}
$$

練習 21

(1) $x^2 + (2y + 1)x + y(y + 1) = (x + y)(x + y + 1)$

(2) $3x^2 + 5xy + 2y^2 - 2x - y - 1 = 3x^2 + (5y - 2)x + (y - 1)(2y + 1) = (x + y - 1)(3x + 2y + 1)$

練習 22

$$
\begin{aligned}
a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2) &= (-b + c)a^2 + (b + c)(b - c)a + bc(c - b) \\
&= (c - b)(a^2 - (b + c)a + bc) = (c - b)(a - b)(a - c) \\
&= (a - b)(b - c)(c - a)
\end{aligned}
$$

練習 23

(1) $x^4 + 2x^2 + 9 = (x^4 + 6x^2 + 9) - 4x^2 = (x^2 + 3)^2 - 4x^2 = (x^2 + 2x + 3)(x^2 - 2x + 3)$

(2) $4x^4 + y^4 = (2x^2 + y^2)^2 - 4x^2y^2 = (2x^2 + y^2 + 2xy)(2x^2 + y^2 - 2xy)$

節末問題

1.

(1) $x^3yz \times \left(-2xy^2z^3 \right)^2 = x^3yz \cdot 4x^2y^4z^6 = 4x^5y^5z^7$

(2) $5ab^2 \times (-0.1ab) \times (-2ab) = 5ab^2 \times \left(-\dfrac{1}{10}ab \right) \times (-2ab) = a^3b^4$

2.

(1) $\left(a-b+2c \right)^2 = a^2 + (-b)^2 + (x)^2 + 2a(-b) + 2(-b)2c + 2\cdot 2c\cdot a$

$\qquad\qquad\quad = a^2 + b^2 + 4c^2 - 2ab - 4bc + 4ca$

(2) $\left(2a-3b \right)^3 = 8a^3 - 36a^2b + 54ab^2 - 27b^3$

(3) $\left(a^2+b^2 \right)\left(a-b \right)\left(a+b \right) = \left(a^2+b^2 \right)\left(a^2-b^2 \right) = a^4 - b^4$

(4) $\left(x^2-x-1 \right)\left(x^2+x+1 \right) = x^4 - \left(x+1 \right)^2 = x^4 - x^2 - 2x - 1$

(5) $\left(a+2b-c \right)\left(a-2b+c \right) = \left(a-c \right)^2 - 4b^2 = a^2 - 2ac + c^2 - 4b^2$

(6) $\left(x-3 \right)\left(x-2 \right)\left(x+2 \right)\left(x+3 \right) = \left(x^2-5x+6 \right)\left(x^2+5x+6 \right)$

$\qquad\qquad\qquad\qquad = \left(x^2+6 \right)^2 - 25x^2 = x^4 + 12x^2 + 36 - 25x^2 = x^4 - 13x^2 + 36$

3.

(1) $3\left(x+2 \right)^2 - 7\left(x+2 \right) - 6 = \left(x+2-3 \right)\left(3\left(x+2 \right)+2 \right) = \left(x-1 \right)\left(3x+8 \right)$

(2) $\left(a+2b \right)\left(a+2b-1 \right) - 6 = \left(a+2b \right)^2 - \left(a+2b \right) - 6 = \left(a+2b-3 \right)\left(a+2b+2 \right)$

(3) $\left(x^2-x \right)^2 - 6\left(x^2-x \right) = \left(x^2-x \right)\left(x^2-x-6 \right) = x\left(x-1 \right)\left(x+2 \right)\left(x-3 \right)$

(4) $y^4z^3 - 64x^3y = y\left(y^3z^3 - 64x^3 \right) = y\left(yz-4x \right)\left(y^2z^2 + 4xyz + 16x^2 \right)$

(5) $x^4 - ax^2 + a - 1 = \left(x^2-a+1 \right)\left(x^2-1 \right) = \left(x+1 \right)\left(x-1 \right)\left(x^2-a+1 \right)$

(6) $x^2y^2 - x^2 - y^2 + 1 = x^2\left(y^2-1 \right) - \left(y^2-1 \right) = \left(x+1 \right)\left(x-1 \right)\left(y+1 \right)\left(y-1 \right)$

4.

(1) $x^2 + 2x - y^2 - 4y - 3 = x^2 + 2x - \left(y+1 \right)\left(y+3 \right) = \left(x-y-1 \right)\left(x+y+3 \right)$

(2) $2x^2 + xy - 6y^2 - x + 19y - 15 = 2x^2 + \left(y-1 \right)x - \left(2y-3 \right)\left(3y-5 \right) = \left(2x-3y+5 \right)\left(x+2y-3 \right)$

(3) $bc\left(b-c \right) + ca\left(c-a \right) + ab\left(a-b \right) = \left(b-c \right)a^2 - \left(b^2-c^2 \right)a + bc\left(b-c \right)$

$\qquad\qquad\qquad\qquad = \left(b-c \right)\left(a^2 - \left(b+c \right)a + bc \right) = \left(a-b \right)\left(b-c \right)\left(a-c \right)$

$\qquad\qquad\text{または} = -(a-b)(b-c)(c-a)$

(4) $x^4 + x^2y^2 + y^4 = \left(x^2+y^2 \right)^2 - x^2y^2 = \left(x^2+y^2+xy \right)\left(x^2+y^2-xy \right)$

$\qquad\qquad = \left(x^2+xy+y^2 \right)\left(x^2-xy+y^2 \right)$