

1 章 数と式 解答

2 節 整式の除法と分数式

練習 1

(1) $(3x^2 - 7x + 1) \div (x - 1)$

$$\begin{array}{r} 3x - 4 \\ x - 1 \overline{) 3x^2 - 7x + 1} \\ \underline{3x^2 - 3x} \\ -4x + 1 \\ \underline{-4x + 4} \\ -3 \end{array}$$

商 $3x - 4$, 余り -3

(2) $(2x^3 - 6x + 4) \div (x + 2)$

$$\begin{array}{r} 2x^2 - 4x + 2 \\ x + 2 \overline{) 2x^3 - 6x + 4} \\ \underline{2x^3 + 4x^2} \\ -4x^2 - 6x \\ \underline{-4x^2 - 8x} \\ 2x + 4 \\ \underline{2x + 4} \\ 0 \end{array}$$

商 $2x^2 - 4x + 2$, 余り 0

(3) $(4x^3 - 9x^2 + 7x) \div (x^2 - 2x + 3)$

$$\begin{array}{r} 4x - 1 \\ x^2 - 2x + 3 \overline{) 4x^3 - 9x^2 + 7x} \\ \underline{4x^3 - 8x^2 + 12x} \\ -x^2 - 5x \\ \underline{-x^2 + 2x - 3} \\ -7x + 3 \end{array}$$

商 $4x - 1$, 余り $-7x + 3$

(4) $(x^3 - 3x^2 - 6) \div (x^2 - 1)$

$$\begin{array}{r} x - 3 \\ x^2 - 1 \overline{) x^3 - 3x^2 - 6} \\ \underline{x^3 - x} \\ -3x^2 + x - 6 \\ \underline{-3x^2 + 3} \\ x - 9 \end{array}$$

商 $x - 3$, 余り $x - 9$

練習 2

$$3x^3 - 4x^2 - 6x + 7 = B(3x - 1) - x + 5$$

$$\begin{array}{r} x^2 - x - 2 \\ 3x - 1 \overline{) 3x^3 - 4x^2 - 5x + 2} \\ \underline{3x^3 - x^2} \\ -3x^2 - 5x \\ \underline{-3x^2 + x} \\ -6x + 2 \\ \underline{-6x + 2} \\ 0 \end{array}$$

$$B = x^2 - x - 2$$

練習 3

$$\begin{array}{r} x^2 + 2xy - 3y^2 \\ x - 2y \overline{) x^3 - 7xy^2 + 6y^3} \\ \underline{x^3 - 2x^2y} \\ 2x^2y - 7xy^2 \\ \underline{2x^2y - 4xy^2} \\ -3xy^2 + 6y^3 \\ \underline{-3xy^2 + 6y^3} \\ 0 \end{array}$$

$$\text{商 } x^2 + 2xy - 3y^2, \text{ 余り } 0$$

練習 4

(1) 最大公約数 $3xyz$, 最小公倍数 $6x^2y^2z^3$

(2) 最大公約数 ac , 最小公倍数 $a^2b^2c^3$

練習 5

(1) $x^2 - 3x = x(x - 3)$ $x^2 - 9 = (x + 3)(x - 3)$

最大公約数 $x - 3$, 最小公倍数 $x(x + 3)(x - 3)$

(2) $x^2 + x + 1$ $x^3 - 1 = (x - 1)(x^2 + x + 1)$

最大公約数 $x^2 + x + 1$, 最小公倍数 $(x - 1)(x^2 + x + 1)$

練習 6

(1) $\frac{10a^2b}{15a^3b^2} = \frac{2}{3ab}$

(2) $\frac{x^2 - 1}{x^2 - x - 2} = \frac{(x + 1)(x - 1)}{(x - 2)(x + 1)} = \frac{x - 1}{x - 2}$

(3) $\frac{2x^2 + 7x - 4}{x^2 + 2x - 8} = \frac{(x + 4)(2x - 1)}{(x + 4)(x - 2)} = \frac{2x - 1}{x - 2}$

(4) $\frac{x^2 - 2x + 1}{x^3 - 1} = \frac{(x - 1)^2}{(x - 1)(x^2 + x + 1)} = \frac{x - 1}{x^2 + x + 1}$

練習 7

$$(1) \quad (x-3) \times \frac{3x}{x^2-6x+9} = \frac{3x(x-3)}{(x-3)^2} = \frac{3x}{x-3}$$

$$(2) \quad \frac{x^2-16}{x^2+1} \times \frac{2x^2+2}{x^2-x-12} = \frac{(x+4)(x-4) \cdot 2(x^2+1)}{(x^2+1)(x-4)(x+3)} = \frac{2x+8}{x+3}$$

$$(3) \quad (x^2-1) \div \frac{x+1}{x-1} = (x+1)(x-1) \cdot \frac{x-1}{x+1} = (x-1)^2 = x^2-2x+1$$

$$(4) \quad \frac{x^2+4x+4}{x^2-2x} \div \frac{x^2+6x+8}{x^2+4x} = \frac{(x+2)^2}{x(x-2)} \cdot \frac{x(x+4)}{(x+2)(x+4)} = \frac{x+2}{x-2}$$

練習 8

$$(1) \quad \frac{x-3}{x^2-4} + \frac{1}{x^2-4} = \frac{x-2}{x^2-4} = \frac{x-2}{(x+2)(x-2)} = \frac{1}{x+2} \quad (2) \quad \frac{x^2}{x-5} - \frac{5x}{x-5} = \frac{x(x-5)}{x-5} = x$$

$$(3) \quad \frac{x^2}{x^2-9} - \frac{6x+9}{9-x^2} = \frac{x^2+6x+9}{x^2-9} = \frac{(x+3)^2}{(x+3)(x-3)} = \frac{x+3}{x-3}$$

練習 9

$$(1) \quad \frac{2}{x+2} + \frac{1}{x-1} = \frac{2(x-1)+x+2}{(x+2)(x-1)} = \frac{3x}{(x+2)(x-1)} \quad (2) \quad \frac{1}{x} - \frac{1-2x}{x^2} = \frac{x-1+2x}{x^2} = \frac{3x-1}{x^2}$$

$$(3) \quad x - \frac{2}{x-3} = \frac{x(x-3)-2}{x-3} = \frac{x^2-3x-2}{x-3}$$

練習 10

$$(1) \quad \frac{x-3}{x^2-x} + \frac{4}{x^2-1} = \frac{x-3}{x(x-1)} + \frac{4}{(x+1)(x-1)} = \frac{(x-3)(x+1)+4x}{x(x+1)(x-1)} = \frac{x^2+2x-3}{x(x+1)(x-1)}$$

$$= \frac{(x+3)(x-1)}{x(x+1)(x-1)} = \frac{x+3}{x(x+1)}$$

$$(2) \quad \frac{x+1}{x^2-3x} - \frac{x-1}{x^2+3x} = \frac{(x+1)(x+3)-(x-1)(x-3)}{x(x-3)(x+3)}$$

$$= \frac{x^2+4x+3-x^2+4x-3}{x(x-3)(x+3)} = \frac{8x}{x(x-3)(x+3)} = \frac{8}{(x-3)(x+3)}$$

$$(3) \quad \frac{1}{x+1} + \frac{3x}{x^3+1} = \frac{x^2-x+1+3x}{(x+1)(x^2-x+1)} = \frac{(x+1)^2}{(x+1)(x^2-x+1)} = \frac{x+1}{x^2-x+1}$$

$$(4) \quad \frac{x+5}{x^2+x-2} - \frac{x-3}{x^2-3x+1} = \frac{x+5}{(x+2)(x-1)} - \frac{x-3}{(x-1)(x-2)}$$

$$= \frac{(x+5)(x-2)-(x-3)(x+2)}{(x-1)(x+2)(x-2)} = \frac{x^2+3x-10-x^2+x+6}{(x-1)(x+2)(x-2)}$$

$$= \frac{4x-4}{(x-1)(x+2)(x-2)} = \frac{4}{(x+2)(x-2)}$$

練習 11

$$(1) \quad \frac{x - \frac{1}{x}}{1 + \frac{1}{x}} = \frac{x^2 - 1}{x + 1} = \frac{(x+1)(x-1)}{x+1} = x-1$$

$$(2) \quad \frac{1 - \frac{x-1}{x+1}}{1 + \frac{x-1}{x+1}} = \frac{x+1 - x+1}{x+1 + x-1} = \frac{2}{2x} = \frac{1}{x} \quad (3) \quad \frac{1}{1 - \frac{1}{1 - \frac{1}{x}}} = \frac{1}{1 - \frac{x}{x-1}} = \frac{x-1}{x-1-x} = 1-x$$

練習 12

$$(1) \quad \frac{3x+1}{x-2} = 3 + \frac{7}{x-2} \quad \begin{array}{r} 3 \\ x-2 \overline{) 3x+1} \\ \underline{3x-6} \\ 7 \end{array}$$

$$(2) \quad \frac{x^2-8}{x+3} = x-3 + \frac{1}{x+3} \quad \begin{array}{r} x-3 \\ x-3 \overline{) x^2-8} \\ \underline{x^2+3x} \\ -3x-8 \\ \underline{-3x-9} \\ 1 \end{array}$$

$$(3) \quad \frac{x^3-4x^2+x-5}{x^2+1} = x-4 - \frac{1}{x^2+1} \quad \begin{array}{r} x-4 \\ x^2+1 \overline{) x^3-4x^2+x-5} \\ \underline{x^3 \quad \quad +x} \\ -4x^2 \quad -5 \\ \underline{-4x^2 \quad -4} \\ -1 \end{array}$$

節末問題

1.

$$(1) \quad A = (3x-1)(x^2+2x-1)-4 = 3x^3+6x^2-3x-x^2-2x+1-4 = 3x^3+5x^2-5x-3$$

$$(2) \quad B = (2x^3-3x^2-6x+7+x-1) \div (2x+3) = (2x^3-3x^2-5x+6) \div (2x+3)$$

$$\begin{array}{r} x^2-3x+2 \\ 2x+3 \overline{) 2x^3-3x^2-5x+6} \\ \underline{2x^3+3x^2} \\ -6x^2-5x \\ \underline{-6x^2-9x} \\ 4x+6 \\ \underline{4x+6} \\ 0 \end{array} \quad \therefore B = x^2-3x+2$$

$$(3) \quad P(x) = Q(x)(x+1)+4$$

$$Q(x) = (x+1)(x-2)+3$$

$$\begin{aligned} \therefore P(x) &= (x+1)^2(x-2)+3(x+1)+4 = (x^2+2x+1)(x-2)+3x+3+4 \\ &= x^3-2x^2+2x^2-4x+x-2+3x+7 = x^3+5 \end{aligned}$$

2. 2つの整式は最大公約数 x^2-x+1 で割り切れる

$$\begin{array}{r} x+2 \\ x^2-x+1 \overline{) x^3+x^2-x+a} \\ \underline{x^3-x^2+x} \\ 2x^2-2x+a \\ \underline{2x^2-2x+2} \\ a-2 \end{array} \qquad \begin{array}{r} 2x-1 \\ x^2-x+1 \overline{) 2x^3-3x^2+3x-b} \\ \underline{2x^3-2x^2+2x} \\ -x^2+x-b \\ \underline{-x^2+x-1} \\ -b+1 \end{array}$$

割り切れるから $a-2=0$ より $a=2$,

$-b+1=0$ より $b=1$,

このとき、最小公倍数は $(x+2)(2x-1)(x^2-x+1)$

3.

$$(1) \quad \frac{1}{x-a} + \frac{x}{a(a-x)} = \frac{a-x}{a(x-a)} = -\frac{1}{a}$$

$$\begin{aligned} (2) \quad \frac{1}{a+2b} - \frac{4b}{4b^2-a^2} &= \frac{1}{a+2b} - \frac{4b}{(2b+a)(2b-a)} \\ &= \frac{2b-a-4b}{(a+2b)(2b-a)} = \frac{-(a+2b)}{(a+2b)(2b-a)} = \frac{1}{a-2b} \end{aligned}$$

$$(3) \quad \frac{1}{x-2} - \frac{1}{x+2} - \frac{4}{x^2+4} = \frac{x+2-x-2}{x^2-4} - \frac{4}{x^2+4} = \frac{4(x^2+4)-4(x^2-4)}{x^4-16} = \frac{32}{x^4-16}$$

$$(4) \quad \left(\frac{x+1}{x} - \frac{x}{x+1} \right) \div \left(2 + \frac{1}{x} \right) = \frac{(x+1)^2 - x^2}{x(x+1)} \cdot \frac{x}{2x+1} = \frac{2x+1}{(x+1)(2x+1)} = \frac{1}{x+1}$$

$$(5) \quad \frac{x^2+x+1}{x^2+x} \times \frac{x^2+2x+1}{x^3-1} \div \frac{x+1}{2x-2} = \frac{x^2+x+1}{x(x+1)} \cdot \frac{(x+1)^2}{(x-1)(x^2+x+1)} \cdot \frac{2(x-1)}{x+1} = \frac{2}{x}$$