１章　数と式　解答

３節　数

練習１

$\dfrac{22}{7}=3.\dot{1}4285\dot{7}$

練習２

(1) $\left|-\dfrac{1}{2}\right|=\dfrac{1}{2}$　　(2) $|5-7|=|-2|=2$

(3) $|2-\sqrt{3}|=2-\sqrt{3}$　　(4) $|3-\pi|=\pi-3$

練習３

(1) $|-1|+|-4|=1+4=5$

(2) $|-1|+|1-4|=|-3|=3$

(3) $|2-1|+|2-4|=|1|+|-2|=3$

(4) $|\pi-1|+|\pi-4|=\pi-1-(\pi-4)=3$

練習４

(1) $\mathrm{A}(-3),\ \mathrm{B}(2)$

$\mathrm{AB}=|-3-2|=|-5|=5$

(2) $\mathrm{A}(-2),\ \mathrm{B}(-5)$

$\mathrm{AB}=|-2-(-5)|=|3|=3$

練習５

(1) $\sqrt{7^2}=7$　　(2) $\sqrt{(-7)^2}=|-7|=7$　　(3) $\sqrt{\left(1-\sqrt{3}\right)^2}=|1-\sqrt{3}|=\sqrt{3}-1$

練習６

(1) $\sqrt{28}=2\sqrt{7}$　　(2) $\sqrt{1000}=10\sqrt{10}$　　(3) $\sqrt{\dfrac{24}{25}}=\dfrac{2\sqrt{6}}{5}$

練習 7

(1) $\sqrt{54}-\sqrt{24}+\sqrt{6}=3\sqrt{6}-2\sqrt{6}+\sqrt{6}=2\sqrt{6}$

(2) $\sqrt{27}+\sqrt{\dfrac{3}{4}}=3\sqrt{3}+\dfrac{\sqrt{3}}{2}=\dfrac{7\sqrt{3}}{2}$

(3) $\left(5+2\sqrt{2}\right)\left(2-3\sqrt{2}\right)=10-15\sqrt{2}+4\sqrt{2}-6.2=-2-11\sqrt{2}$

(4) $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{20}-\sqrt{12}\right)=\sqrt{100}-\sqrt{60}+\sqrt{60}-\sqrt{36}=10-6=4$

(5) $\left(\sqrt{2}+3\sqrt{3}\right)^2=2+4\sqrt{6}+12=14+4\sqrt{6}$

(6) $\left(2\sqrt{2}-\sqrt{3}\right)^3=\left(2\sqrt{2}\right)^3-3\left(2\sqrt{2}\right)^2\sqrt{3}+3\cdot2\sqrt{2}\left(\sqrt{3}\right)^2-\left(\sqrt{3}\right)^2$

$=16\sqrt{2}-24\sqrt{3}+18\sqrt{2}-3\sqrt{3}=34\sqrt{2}-27\sqrt{3}$

練習 8

(1) $\dfrac{1}{\sqrt{7}+\sqrt{3}}=\dfrac{\sqrt{7}-\sqrt{3}}{\left(\sqrt{7}+\sqrt{3}\right)\left(\sqrt{7}-\sqrt{3}\right)}=\dfrac{\sqrt{7}-\sqrt{3}}{7-3}=\dfrac{\sqrt{7}-\sqrt{3}}{4}$

(2) $\dfrac{\sqrt{2}}{2-\sqrt{2}}=\dfrac{\sqrt{2}\left(2+\sqrt{2}\right)}{\left(2-\sqrt{2}\right)\left(2+\sqrt{2}\right)}=\dfrac{2\sqrt{2}+2}{4-2}=\dfrac{2+2\sqrt{2}}{2}=1+\sqrt{2}$

(3) $\dfrac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}=\dfrac{\left(\sqrt{6}-\sqrt{5}\right)^2}{\left(\sqrt{6}+\sqrt{5}\right)\left(\sqrt{6}-\sqrt{5}\right)}=\dfrac{6-2\sqrt{30}+5}{6-5}=11-2\sqrt{30}$

練習 9 $x=\dfrac{2\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}=\sqrt{5}+\sqrt{3}$

$y=\dfrac{2\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}=\sqrt{5}-\sqrt{3}$

(1) $x+y=\left(\sqrt{5}+\sqrt{3}\right)+\left(\sqrt{5}-\sqrt{3}\right)=2\sqrt{5}$

(2) $xy=\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)=2$

(3) $x^2+y^2=\left(\sqrt{5}+\sqrt{3}\right)^2+\left(\sqrt{5}-\sqrt{3}\right)^2=8+2\sqrt{15}+8-2\sqrt{15}=16$

別解 $x^2+y^2=\left(x+y\right)^2-2xy=\left(2\sqrt{5}\right)^2-2\cdot2=16$

練習 10

(1) $-1+\sqrt{3}i$

実部　−1，虚部　$\sqrt{3}$

(2) $4-i$

実部　4，虚部　−1

(3) $5i$

実部　0，虚部　5

(4) 2

実部　−2，虚部　0

練習 11

(1) $(a+3)+(2a+b)i=0$

$a+3,\ 2a+b$ は実数であるから

$$\begin{cases} a+3=0 \\ 2a+b=0 \end{cases}$$

$a=-3,\ b=6$

(2) $(a+4b)+(3a-2)i=9-i$

$a+4b,\ 3a-2b$ は実数であるから

$$\begin{cases} a+4b=9 \\ 3a-2b=-1 \end{cases}$$

$$\begin{array}{r} -)\ 3a+12b=27 \\ \hline -14b=-28 \end{array}$$

$a=-1,\ b=2$

練習 12

(1) $(3+i)+(1-4i)=4-3i$

(2) $(2-i)-(5+2i)=-3-3i$

(3) $(1+2i)(1-2i)=1-4i^2=5$

(4) $(1-3i)(3+2i)=3+2i-9i-6i^2=9-7i$

(5) $(1+i)^2=1+2i+i^2=2i$

(6) $(1+i)(i^2+i^3)=i^2+i^3+i^3+i^4=-1-i-i+1=2i$

練習 13

(1) 1－3i (2) 5＋4i (3) $-\sqrt{2}\,i$ (4) －3

練習 14

(1) $\dfrac{i}{2-i}=\dfrac{i(2+i)}{(2-i)(2+i)}=\dfrac{2i+i^2}{4-i^2}=\dfrac{-1+2i}{5}$

(2) $\dfrac{1-i}{1+i}=\dfrac{(1-i)^2}{(1+i)(1-i)}=\dfrac{1-2i+i^2}{1-i^2}=\dfrac{-2i}{2}=-i$

(3) $\dfrac{2+i}{1+3i}=\dfrac{(2+i)(1-3i)}{(1+3i)(1-3i)}=\dfrac{2-6i+i-3i^2}{1+9}=\dfrac{5-5i}{10}=\dfrac{1-i}{2}$

(4) $\dfrac{1}{i}=\dfrac{-i}{i\cdot(-i)}=-i$

練習 15

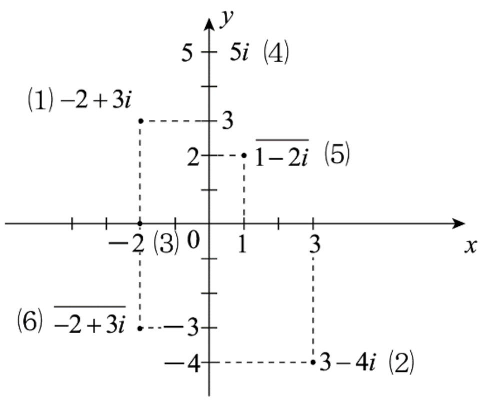
(1) $\sqrt{-2}\sqrt{-8}=\sqrt{2}\,i\,2\sqrt{2}\,i=4i^2=-4$

(2) $\dfrac{\sqrt{-6}}{\sqrt{-27}}=\dfrac{\sqrt{6}\,i}{3\sqrt{3}\,i}=\dfrac{\sqrt{2}}{3}$

(3) $\dfrac{\sqrt{-3}}{\sqrt{2}}=\dfrac{\sqrt{3}\,i}{\sqrt{2}}=\dfrac{\sqrt{6}}{2}i$

(4) $\dfrac{\sqrt{45}}{\sqrt{-5}}=\dfrac{\sqrt{45}}{\sqrt{5}\,i}=\dfrac{\sqrt{9}}{i}=-3i$

練習 16



練習 17

⑴　$|-3+4i|=\sqrt{(-3)^2+4^2}=5$

⑵　$|2-3i|=\sqrt{2^2+(-3)^2}=\sqrt{13}$

⑶　$|5|=5$

⑷　$|4i|=4$

練習 18

⑴　$|(4+3i)(3-4i)|=|4+3i||3-4i|=\sqrt{4^2+3^2}\sqrt{3^2+(-4)^2}=25$

⑵　$\left|(1-3i)\left(2+\sqrt{6}\,i\right)\right|=|1-3i|\left|2+\sqrt{6}\,i\right|=\sqrt{1^2+(-3)^2}\sqrt{2^2+6}$

$$=2\cdot 2\sqrt{10}=4\sqrt{10}$$

⑶　$\left|\dfrac{2-i}{3+i}\right|=\dfrac{|2-i|}{|3+i|}=\dfrac{\sqrt{2^2+(-1)^2}}{\sqrt{3^2+1^2}}=\dfrac{\sqrt{5}}{\sqrt{10}}=\dfrac{1}{\sqrt{2}}$

⑷　$\left|\dfrac{1-\sqrt{3}\,i}{i(1+i)}\right|=\dfrac{\left|1-\sqrt{3}\,i\right|}{|i||1+i|}=\dfrac{\sqrt{1^2+\left(-\sqrt{3}\right)^2}}{1\cdot\sqrt{1^2+1^2}}=\dfrac{2}{\sqrt{2}}=\sqrt{2}$

練習 19

〔2〕の証明

$$\overline{\alpha-\beta}=\overline{(a+bi)-(c+di)}=\overline{(a-c)+(b-d)i}=(a-c)-(b-d)i$$
$$\overline{\alpha}-\overline{\beta}=\overline{a+bi}-\overline{(c+di)}=a-bi-(c-di)=(a-c)-(b-d)i$$

〔3〕の証明

$$\overline{\alpha\beta}=\overline{(a+bi)(c+di)}=\overline{(ac-bd)+(ad+bc)i}=(ac-bd)-(ad+bc)i$$
$$\overline{\alpha}\,\overline{\beta}=\overline{(a+bi)}\,\overline{(c+di)}=(a-bi)(c-di)=(ac-bd)-(ad-bc)i$$

〔4〕の証明

$$\frac{\alpha}{\beta}=\frac{a+bi}{c+di}=\frac{(a+bi)(c-di)}{(c+di)(c-di)}=\frac{(ac+bd)-(ad-bc)i}{c^2+d^2}$$
$$\overline{\left(\frac{\alpha}{\beta}\right)}=\frac{(ac+bd)+(ad-bc)i}{c^2+d^2}$$
$$\frac{\overline{\alpha}}{\overline{\beta}}=\frac{a-bi}{c-di}=\frac{(a-bi)(c+di)}{(c-di)(c+di)}=\frac{(ac+bd)+(ad-bc)i}{c^2+d^2}$$

よって，〔2〕,〔3〕,〔4〕は成り立つ。

節末問題

１．

⑴ $\sqrt{2}-\dfrac{\sqrt{32}}{3}+\dfrac{1}{\sqrt{8}}=\dfrac{12-16+3}{6\sqrt{2}}=\dfrac{-1}{6\sqrt{2}}=-\dfrac{\sqrt{12}}{12}$

⑵ $\left(3\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}-2\sqrt{3}\right)=15-6\sqrt{15}-\sqrt{15}+6=21-7\sqrt{15}$

⑶ $\left(1+\sqrt{2}+\sqrt{3}\right)^2=\left(1+\sqrt{2}\right)^2+2\left(1+\sqrt{2}\right)\sqrt{3}+3$

$=1+2\sqrt{2}+2+2\sqrt{3}+2\sqrt{6}+3=6+2\sqrt{2}+2\sqrt{3}+2\sqrt{6}$

⑷ $\sqrt{\left(1-\sqrt{5}\right)^2}+\sqrt{\left(3-\sqrt{5}\right)^2}=\left|1-\sqrt{5}\right|+\left|3-\sqrt{5}\right|=\sqrt{5}-1+3-\sqrt{5}=2$

⑸ $\left(\dfrac{1}{\sqrt{3}+\sqrt{2}}\right)^2=\left(\dfrac{\sqrt{3}-\sqrt{2}}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}\right)^2=\left(\sqrt{3}-\sqrt{2}\right)^2=3-2\sqrt{6}+2=5-2\sqrt{6}$

(6) $\dfrac{1}{2+\sqrt{3}}-\dfrac{1}{\sqrt{5}-2}=\dfrac{2-\sqrt{3}}{1}-\dfrac{\sqrt{5}+2}{1}=-\sqrt{3}-\sqrt{5}$

２．

$-\dfrac{1}{\sqrt{1}+\sqrt{2}}+\dfrac{1}{\sqrt{2}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{4}}+\dfrac{1}{\sqrt{4}+\sqrt{5}}$

$=-\dfrac{\sqrt{1}-\sqrt{2}}{-1}+\dfrac{\sqrt{2}-\sqrt{3}}{-1}+\dfrac{\sqrt{3}-\sqrt{4}}{-1}+\dfrac{\sqrt{4}-\sqrt{5}}{-1}$

$=\left(\sqrt{2}-1\right)+\left(\sqrt{3}-\sqrt{2}\right)+\left(\sqrt{4}-\sqrt{3}\right)+\left(\sqrt{5}-\sqrt{4}\right)$

$=\sqrt{5}-1$

３． $x=\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$， $y=\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ のとき

$x=\dfrac{\left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}=\dfrac{8+2\sqrt{5}}{2}=4+\sqrt{15}$

$y=\dfrac{\left(\sqrt{5}-\sqrt{3}\right)^2}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}=\dfrac{8-2\sqrt{5}}{2}=4-\sqrt{15}$

⑴ $x+y=\left(4+\sqrt{15}\right)+\left(4-\sqrt{15}\right)=8$

⑵ $xy=1$

⑶ $x^2+y^2=\left(4+\sqrt{15}\right)^2+\left(4-\sqrt{15}\right)^2=16+8\sqrt{15}+15+16-8\sqrt{15}+15=62$

⑷ $x^3+y^3=(x+y)\left(x^2-xy+y^2\right)=8\cdot(62-1)=8\cdot 61=488$

⑸ $\dfrac{y}{x}+\dfrac{x}{y}=\dfrac{x^2+y^2}{xy}=62$

⑹ $\dfrac{y+1}{x-1}+\dfrac{x+1}{y-1}=\dfrac{y^2-1+x^2-1}{(x-1)(y-1)}=\dfrac{x^2+y^2-2}{xy-(x+y)+1}=\dfrac{62-2}{1-8+1}=\dfrac{60}{-6}=-10$

別解

⑵ $x^2+y^2=(x+y)^2-2xy=8^2-2\cdot 1=62$

⑶ $x^3+y^3=(x+y)^3-3xy(x+y)=8^3-3\cdot 1\cdot 8=488$

4． $2<\sqrt{5}<3(\sqrt{5}=2.236\cdots)$

⑴ $a=2$ ⑵ $b=\sqrt{5}-2$

⑶ $a-\dfrac{1}{b}=2-\dfrac{1}{\sqrt{5}-2}=\dfrac{2\sqrt{5}-4-1}{\sqrt{5}-2}=\dfrac{2\sqrt{5}-5}{\sqrt{5}-2}=\left(2\sqrt{5}-5\right)\left(\sqrt{5}+2\right)$

$=10+4\sqrt{5}-5\sqrt{5}-10=-\sqrt{5}$

5．

⑴ $|x|+|x-4|=-x-x+4=-2x+4$

⑵ $|x|+|x-4|=x-x+4=4$

⑶ $|x|+|x-4|=x+x-4=2x-4$

6．

⑴ $(2+i)(x+yi)=-5+5i$

$(2x-y)+(x+2y)i=-5+5i$

$2x-y,\ x+2y$は実数であるから

$\begin{cases}2x-y=-5 & ①\\ x+2y=5 & ②\end{cases}$

①$-2\times$② $-5y=15$

$x=-1,\ y-3$

⑵ $\dfrac{x+7i}{1+yi}=4-i$

$x+7i=(4-i)(1+yi)=4+4yi-i+y$

$=(y+4)+i(4y-1)$

$y+4,\ 4y-1$は実数であるから

$\begin{cases}x=y+4\\ 7=4y-1\end{cases}\ \rightarrow x=6,\ y-2$

7．

⑴ $\left(\sqrt{-5}+\sqrt{2}\right)\left(\sqrt{8}-\sqrt{5}\right)=\left(\sqrt{5}\,i+\sqrt{2}\right)\left(2\sqrt{2}\,i-\sqrt{5}\right)=-2\sqrt{10}-5i+4i-\sqrt{10}=-3\sqrt{10}-i$

⑵ $(1-i)^2=1+3i+3i^2-i^3=1-3i-3+i=-2-2i$

⑶ $i+i^2+i^3+i^4+\dfrac{1}{i}=i-1-i+1-i=-i$

⑷ $5-2i+\overline{5-2i}=5-2i+5+2i=10$

⑸ $\overline{i}(2-2i)=-i(2-2i)=-2-2i$

⑹ $\left(3+\sqrt{2}\,i\right)\overline{\left(3+\sqrt{2}\,i\right)}=\left(3+\sqrt{2}\,i\right)\left(3-\sqrt{2}\,i\right)=9+2=11$

演習

⑴ $\sqrt{8+2\sqrt{15}}=\sqrt{(5+3)+2\sqrt{5\cdot 3}}=\sqrt{5}+\sqrt{3}$

⑵ $\sqrt{4-\sqrt{12}}=\sqrt{4-2\sqrt{3}}=\sqrt{(3+1)-2\sqrt{3\cdot 1}}=\sqrt{3}-1$

⑶ $\sqrt{9+4\sqrt{5}}=\sqrt{9+2\sqrt{20}}=\sqrt{(5+4)+2\sqrt{5\cdot 4}}=\sqrt{5}+\sqrt{4}=\sqrt{5}+2$

⑷ $\sqrt{3-\sqrt{5}}=\sqrt{\dfrac{6-2\sqrt{5}}{2}}=\sqrt{\dfrac{(5+1)+2\sqrt{5\cdot 1}}{\sqrt{2}}}=\dfrac{\sqrt{5}-1}{\sqrt{2}}=\dfrac{\sqrt{10}-\sqrt{2}}{2}$