２章　２次関数とグラフ，方程式・不等式　解答

２節　２次関数とグラフ

練習１

⑴　$f(x)=3x-5$

$f(3)=9-5=4$，　$f(-1)=-3-5=-8$，　$f(a+1)=3a+3-5=3a-2$

⑵　$f(x)=2x^2+x$

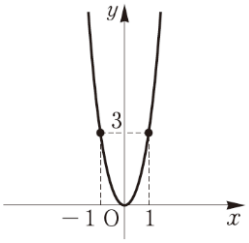
$f(3)=2\cdot 3^2+3=18+3=21$，

$f(-1)=2\cdot(-1)^2+(-1)=2-1=1$，

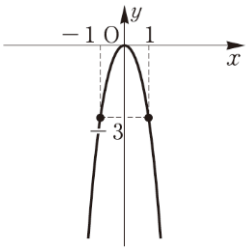
$f(a+1)=2(a+1)^2+(a+1)=2(a^2+2a+1)+a+1=2a^2+5a+3$

練習２

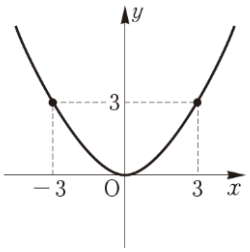
⑴　$y=3x^2$



⑵　$y=-3x^2$



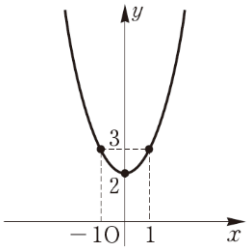
⑶　$y=\frac{1}{3}x^2$



練習３

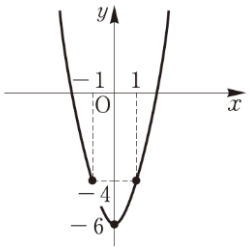
⑴　$y=x^2+2$

頂点　$(0,\ 2)$



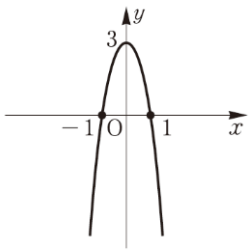
⑵　$y=2x^2-6$

頂点　$(0,\ -6)$



⑶　$y=-3x^2+3$

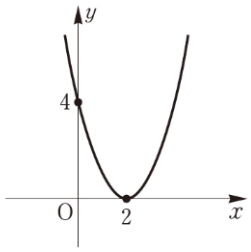
頂点　$(0,\ 3)$



練習 4

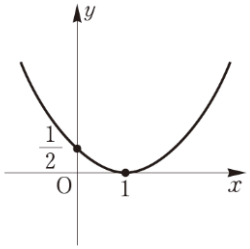
⑴ $y=(x-2)^2$

軸 $x=2$, 頂点 $(2,\ 0)$



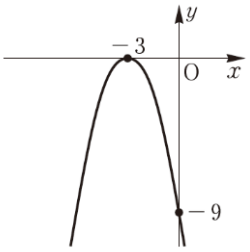
⑵ $y=\frac{1}{2}(x-1)^2$

軸 $x=1$, 頂点 $(1,\ 0)$



⑶ $y=-(x+3)^2$

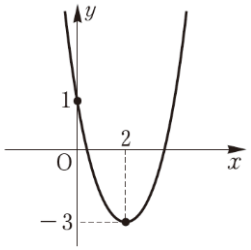
軸 $x=-3$, 頂点 $(-3,\ 0)$



練習 5

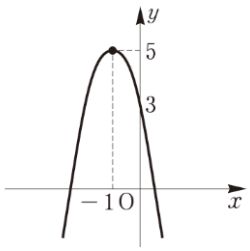
⑴ $y=(x-2)^2-3$

軸 $x=2$, 頂点 $(2,\ -3)$



⑵ $y=-2(x+1)^2+5$

軸 $x=-1$, 頂点 $(-1,\ 5)$



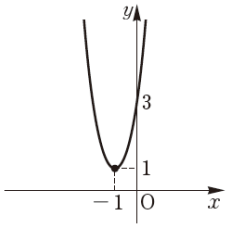
練習 6

$y=3(x-(-4))^2+5=3(x+4)^2+5$

練習 7

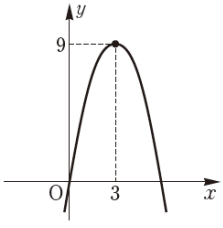
⑴ $y=2x^2+4x+3=2(x^2+2x)+3=2(x+1)^2+1$

軸 $x=-1$, 頂点 $(-1,\ 1)$



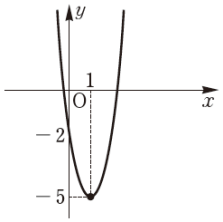
⑵ $y=-x^2+6x=-(x^2-6x)=-(x-3)^2+9$

軸 $x=3$, 頂点 $(3,\ 9)$



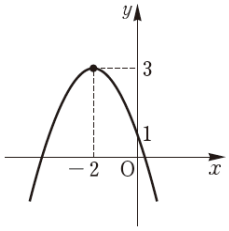
⑶　$y=3x^2-6x-2=3\left(x^2-2x\right)-2=3\left(x-1\right)^2-5$

軸　$x=1$,　頂点　$\left(1,\ -5\right)$



⑷　$y=-\frac{1}{2}x^2-2x+1=-\frac{1}{2}\left(x^2+4x\right)+1=-\frac{1}{2}\left(x+2\right)^2+3$

軸　$x=-2$,　頂点　$\left(-2,\ 3\right)$



練習 8

⑴　$y=2x^2-8x=2\left(x^2-4x\right)=2\left(x-2\right)^2-8$

x 軸方向に-2,　y 軸方向に8

⑵　$y=2x^2+4x-1=2\left(x^2+2x\right)-1=2\left(x+1\right)^2-3$

x 軸方向に-3,　y 軸方向に5

練習 9

⑴　$y=a\left(x-1\right)^2+5$　とおける。

点$\left(-1,\ 1\right)$を通るから

$1=a\left(-1-1\right)^2+5$

$-4=4a$　　$a=-1$

$\therefore\ y=-\left(x-1\right)^2+5=-x^2+2x+4$

⑵　$y=a\left(x-2\right)^2+q$　とおける。

2点$\left(-2,\ 5\right)$, $\left(4,\ -1\right)$を通るから

$\left(-2,\ 5\right)$　$5=a\left(-2-2\right)^2+q$

$\left(4,\ -1\right)$　$-1=a\left(4-2\right)^2+q$

$6=12a$　　$a=\frac{1}{2}$　　$q=-3$

$\therefore\ y=\frac{1}{2}\left(x-2\right)^2-3=\frac{1}{2}x^2-2x-1$

練習 10

⑴　$y=ax^2+bx+1$

$\left(0,\ -2\right)$を通る　$-2=c$　　①

$\left(1,\ \ 3\right)$を通る　$3=a+b+c$　　②

$\left(-2,-6\right)$を通る　$-6=4a-2b+c$　③

①を②に代入

$a+b=5$　　①′

①を③に代入

$4a-2b=-4$　　②′

①′, ②′を解いて

$a=1$　$b=4$　$c=-2$　$\therefore\ y=x^2+4x-2$

⑵ $(-1,\ -8)$を通る　$-8 = a - b + c$　①

$(2,\ 7)$を通る　$7 = 4a + 2b + c$　②

$(3,\ 4)$を通る　$4 = 9a + 3b + c$　③

①－②　$-15 = -3a - 3b$　$5 = a + b$

①－③　$-12 = -8a - 4b$　$-)\ 3 = 2a + b$

$2 = -a$

$a = -2$　$b = 7$　を①に代入　$-8 = -2 - 7 + c$　$c = 1$

$A,\ a = -2,\ b = 7,\ c = 1$

$\therefore\ y = -2x^2 + 7x + 1$

⑶ $\left(1,\ \frac{3}{2}\right)$を通る　$\frac{3}{2} = a + b + c$　①

$(2,\ 2)$を通る　$2 = 4a + 2b + c$　②

$(-2,\ 6)$を通る　$6 = 4a - 2b + c$　③

②－③　$-4 = 4b,\ b = -1$

①に代入して　$a + c = \frac{5}{2}$　①′

②に代入して　$4a + c = 4$　②′

②′－①′　$3a + \frac{3}{2},\ a = \frac{1}{2},\ c = 2$

$\therefore\quad y = \frac{1}{2}x^2 - x + 2$

練習 12

⑴　$y = x^2 + 6x + 7 = (x + 3)^2 - 2$　最大値なし，最小値-2

⑵　$y = -x^2 + 4$　最大値4，最小値なし

⑶　$y = 3x^2 - 2x + 1 = 3\left(x^2 - \frac{2}{3}x\right) + 1 = 3\left(x - \frac{1}{3}\right)^2 + \frac{2}{3}$　最大値なし，最小値$\frac{2}{3}$

⑷　$y = -\frac{1}{2}x^2 + x + 5 = -\frac{1}{2}(x^2 - 2x) + 5 = -\frac{1}{2}(x - 1)^2 + \frac{11}{2}$　最大値$\frac{11}{2}$，最小値なし

練習 13

$y = (x - 1)^2 - 2\quad(-1 \leqq x \leqq 2)$

$x = -1$ のとき最大値 2，$x = 1$のとき 最小値 -2

練習 14

⑴ $y = x^2 - 3 \quad (-2 \leqq x \leqq 1)$

$x = -2$ のとき 最大値1, $x = 0$ のとき最小値-3

⑵ $y = -x^2 - 4x + 2 \quad (-1 \leqq x \leqq 1)$

$= -(x^2 + 4x) + 2 = -(x + 2)^2 + 6$

$x = -1$のとき 最大値5, $x = 1$ のとき最小値-3

節末問題

1． $y = 2x^2 - 12x + 13 = 2(x^2 - 6x) + 13 = 2(x - 3)^2 - 5$

⑴ $y = 2(x + 3)^2 - 5 = 2(x^2 + 6x + 9) - 5 = 2x^2 + 12x + 13$

⑵ $y = -2(x - 3)^2 + 5 = -2(x^2 - 6x + 9) + 5 = -2x^2 + 12x - 13$

⑶ $y = -2(x + 3)^2 + 5 = -2(x^2 + 6x + 9) + 5 = -2x^2 - 12x - 13$

2．

⑴ $y = \frac{1}{2}x^2 - 2x + 1 \quad (-2 \leqq x \leqq 1)$

$= \frac{1}{2}(x^2 - 4x) + 1 = \frac{1}{2}(x - 2)^2 - 1$

最大値 $7\ (x = -2)$　　最小値 $-\frac{1}{2}\ (x = 1)$

⑵ $y = -2x^2 + x \quad (0 \leqq x \leqq 1)$

$= -2\left(x^2 - \frac{1}{2}x\right) = -2\left(x - \frac{1}{4}\right)^2 + \frac{1}{8}$

最大値 $\frac{1}{8}\ \left(x = \frac{1}{4}\right)$　　最小値 $-1\ (x = 1)$

3．

⑴ $y = -a(x - 2)^2 + 9$

$-3 = -a(4 - 2)^2 + 9$

$-12 = -a \cdot 4$

$a = 3 \quad \therefore$

$y = -3(x - 2)^2 + 9$

$= -3(x^2 - 4x + 4) + 9$

$= -3x^2 + 12x - 3$

⑵ $y = a(x - 3)^2 + q$

$(0,\ 3) \qquad 3 = 9a + q$

$(2,\ -1) \quad -) \underline{-1 = a + q}$

$4 = 8a$

$a = \frac{1}{2}$

$-1 = \frac{1}{2} + q \qquad -\frac{3}{2} = q$

$y = \frac{1}{2}(x - 3)^2 - \frac{3}{2} = \frac{1}{2}x^2 - 3x + 3$

⑶ $y = a\left(x + \frac{1}{2}\right)^2$

$(1,\ 9) \qquad 9 = a\left(1 + \frac{2}{2}\right)^2$

$9 = a\left(\frac{9}{4}\right)$

$a = 4$

$y = 4\left(x + \frac{1}{2}\right)^2 = 4x^2 + 4x + 1$

⑷ $y = ax^2 + bx + c$

$(1,\ 3) \qquad 3 = a + b + c \qquad ①$

$(2,\ 6) \qquad 6 = 4a + 2b + c \qquad ②$

$(-1,\ 9) \qquad 9 = a - b + c \qquad ③$

① − ② $\quad -3 = -3a - b$

① − ③ $\quad -6 = 2b \quad b = -3$

$-3 = -3a + 3 \qquad a = 2 \quad c = 4$

$y = 2x^2 - 3x + 4$

⑸　$y=3(x-p)^2+q$

$(-1,\ 3)$　$3=3(-1-p)^2+q$　①

$(1,\ 1)$　$1=3(1-p)^2+q$　②

①－②　$2=3\left(1+2p+p^2-1+2p-p^2\right)=12p$　$p=\dfrac{1}{6}$

$1=3\left(\dfrac{5}{6}\right)^2+q$

$1=\dfrac{25}{12}+q$　$q=-\dfrac{13}{12}$

$y=3\left(x-\dfrac{1}{6}\right)^2-\dfrac{13}{12}=3\left(x^2-\dfrac{x}{3}+\dfrac{1}{36}\right)-\dfrac{13}{12}=3x^2-x-1$

4．$y=a(x-1)^2+b-a$

$a>0$ より $x=1$ で最小値 $b-a=1$　①

$x=3$ で最大値 $9a-6a+b=5$

$3a+b=5$　②

①－②　$-4a=-4$　$a=1,\quad b=2$

5．　$x^2+3(1-x)=\left(x-\dfrac{3}{2}\right)^2+\dfrac{3}{4}$

$x=\dfrac{3}{2},\ y=-\dfrac{1}{2}$のとき最小値$\dfrac{3}{4}$

6．$1:2=(5-x):y$

$y=2(5-x)$　∴ AQ$=10-2(5-x)$

⑴　$S=(5-x)(10-2(5-x))=2x(5-x)=-2x^2+10x$

⑵　$S=-2\left(x^2-5x\right)=-2\left(x-\dfrac{5}{2}\right)^2+\dfrac{25}{2}$　$x=\dfrac{5}{2}$のとき最大値$\dfrac{25}{2}$