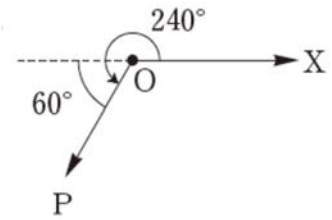
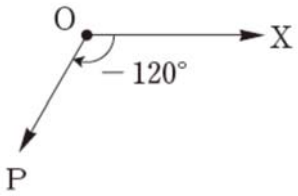
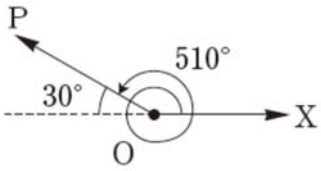
６章　三角関数　解答

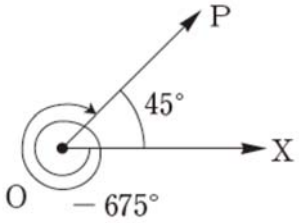
２節　三角関数

練習１









練習２

(1)　$405° = 45° + 360° \times 1$

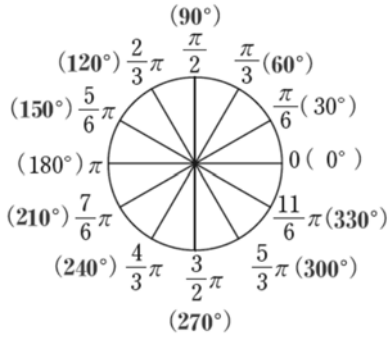
(2)　$840° = 120° + 360° \times 2$

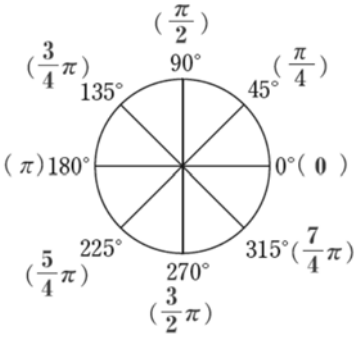
(3)　$-270° = 90° + 360° \times (-1)$

(4)　$1050° = 330° + 360° \times 2$

練習３

(1)　$\frac{2}{3}\pi$　　(2)　$\frac{3}{4}\pi$　　(3)　$\frac{7}{6}\pi$　　(4)　$-\frac{5}{3}\pi\left(=\frac{\pi}{3}\right)$





練習４

$x°$ のとき α ラジアンとすると

$360° = x° = 2\pi = \alpha$

$$x° = \frac{180°}{\pi} \times \alpha$$

⑴ $x° = \dfrac{180°}{\pi} \times \dfrac{3}{4}\pi = 135°$

⑵ $x° = \dfrac{180°}{\pi} \times \left(-\dfrac{4}{3}\pi \right) = -240°$

⑶ $x° = \dfrac{180°}{\pi} \times \dfrac{2}{5}\pi = 72°$

⑷ $x° = \dfrac{180°}{\pi} \times \left(\dfrac{11}{3}\pi \right) = 660°$

練習５

⑴ $l = r\theta$ より $l = 4 \times \dfrac{\pi}{4} = \pi$

$S = \dfrac{1}{2}r^2\theta$ より $S = \dfrac{1}{2} \times 4^2 \times \dfrac{\pi}{4} = 2\pi$

⑵ $l = r\theta$ より $l = 6 \times \dfrac{5}{6}\pi = 5\pi$

$S = \dfrac{1}{2}r^2\theta$ より $S = \dfrac{1}{2} \times 6 \times 5\pi = 15\pi$

練習６

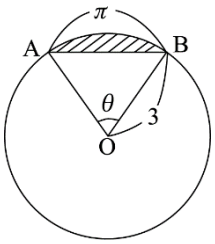
中心角を θ とすると

$6\pi : \pi = 2\pi : \theta$ より $\theta = 2\pi^2 \times \dfrac{\pi}{6} = \dfrac{\pi}{3}$

扇形の OAB の面積は　$\dfrac{1}{2} \times 3^2 \times \dfrac{\pi}{3} = \dfrac{3}{2}\pi$

△OAB の面積は　$\dfrac{1}{2} \times 3 \times 3 \times \sin\dfrac{\pi}{3} = \dfrac{9\sqrt{3}}{4}$

よって，斜線部分の面積は　$\dfrac{3}{2}\pi - \dfrac{9\sqrt{3}}{4}$



練習７

⑴ $\sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

$\cos\dfrac{\pi}{3} = \dfrac{1}{2}$

$\tan\dfrac{\pi}{3} = \sqrt{3}$

⑵ $\sin\left(-\dfrac{5}{4}\pi \right) = \dfrac{1}{\sqrt{2}}$

$\cos\left(-\dfrac{5}{4}\pi \right) = -\dfrac{1}{\sqrt{2}}$

$\tan\left(-\dfrac{5}{4}\pi \right) = -1$

⑶ $\sin\left(\dfrac{19}{6}\pi \right) = -\dfrac{1}{2}$

$\cos\left(\dfrac{19}{6}\pi \right) = -\dfrac{\sqrt{3}}{2}$

$\tan\left(\dfrac{19}{6}\pi \right) = \dfrac{1}{\sqrt{3}}$

⑷ $\sin\left(-\dfrac{\pi}{6} \right) = -\dfrac{1}{2}$

$\cos\left(-\dfrac{\pi}{6} \right) = \dfrac{\sqrt{3}}{2}$

$\tan\left(-\dfrac{\pi}{6} \right) = -\dfrac{1}{\sqrt{3}}$

練習８

⑴　$\sin\theta < 0,\quad \cos\theta < 0$

第 3 象限

⑵　$\cos\theta > 0,\quad \tan\theta < 0$

第 4 象限

練習９

⑴　$\sin^2\theta+\cos^2\theta=1$ より $\sin^2\theta=1-\cos^2\theta=1-\left(\dfrac{3}{4}\right)^2=\dfrac{7}{16}$

$\dfrac{3\pi}{2}<\theta<2\pi$ より $\sin\theta<0$　$\therefore \sin\theta=-\dfrac{\sqrt{7}}{4}$

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ より　$\tan\theta=\dfrac{\left(-\dfrac{\sqrt{7}}{4}\right)}{\left(\dfrac{3}{4}\right)}=-\dfrac{\sqrt{7}}{3}$

⑵　$1+\tan^2\theta=\dfrac{1}{\cos^2\theta}$ より $\dfrac{1}{\cos^2\theta}=1+3^2=10$

$\cos^2\theta=\dfrac{1}{10}$

$\pi<\theta<\dfrac{3}{2}\pi$ より $\cos\theta<0$　$\therefore \cos\theta=-\dfrac{1}{\sqrt{10}}=-\dfrac{\sqrt{10}}{10}$

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ より $\sin\theta=\tan\theta\cos\theta=3\times\left(-\dfrac{1}{\sqrt{10}}\right)=-\dfrac{3}{\sqrt{10}}=-\dfrac{3\sqrt{10}}{10}$

練習 10

$\sin^2\theta+\cos^2\theta=1$ より

$\cos^2\theta=1-\sin^2\theta=1-\left(-\dfrac{\sqrt{6}}{3}\right)^2=\dfrac{1}{3}$

$\cos\theta=\pm\dfrac{\sqrt{3}}{3}$

$\sin\theta=-\dfrac{\sqrt{3}}{6}<0$ だから

$\pi<\theta<\dfrac{3}{2}\pi$ のとき　$\cos\theta=-\dfrac{\sqrt{3}}{3}$

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}=-\dfrac{\sqrt{3}}{6}\times\left(-\dfrac{3}{\sqrt{3}}\right)=\dfrac{1}{2}$

$\dfrac{3}{2}\pi<\theta<2\pi$ のとき $\cos\theta=\dfrac{\sqrt{3}}{3}$

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}=-\dfrac{\sqrt{3}}{6}\times\dfrac{3}{\sqrt{3}}=\dfrac{1}{2}$

練習 11

⑴ $\sin\dfrac{13}{6}\pi=\sin\dfrac{\pi}{6}=\dfrac{1}{2}$

⑵ $\cos\dfrac{8}{3}\pi=\cos\dfrac{2}{3}\pi=-\dfrac{1}{2}\pi$

⑶ $\tan\dfrac{17}{4}\pi=\tan\dfrac{\pi}{4}=1$

練習 12

⑴ $\sin\left(-\dfrac{\pi}{4}\right)=-\sin\dfrac{\pi}{4}=-\dfrac{1}{\sqrt{2}}$

⑵ $\cos\left(-\dfrac{2}{3}\pi\right)=\cos\dfrac{2}{3}\pi=-\dfrac{1}{2}$

⑶ $\tan\left(-\dfrac{5}{6}\pi\right)=-\tan\dfrac{5}{6}\pi=-\left(-\dfrac{1}{\sqrt{3}}\right)=\dfrac{1}{\sqrt{3}}$

練習 13

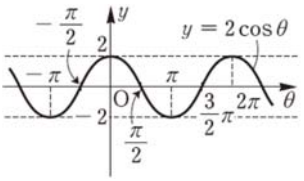
⑴ $\sin\dfrac{6}{5}\pi=\sin\left(\dfrac{1}{5}\pi+\pi\right)=-\sin\dfrac{\pi}{5}=-a$

⑵ $\cos\dfrac{3}{10}\pi=\cos\left(\dfrac{\pi}{2}-\dfrac{2}{10}\pi\right)=\sin\dfrac{\pi}{5}=a$

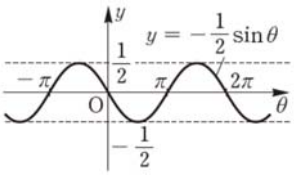
⑶ $\sin\left(-\dfrac{16}{5}\pi\right)=-\sin\dfrac{16}{5}\pi=-\sin\left(2\pi+\dfrac{6}{5}\pi\right)=\sin\dfrac{6}{5}\pi=\sin\left(\dfrac{1}{5}\pi+\pi\right)=\sin\dfrac{\pi}{5}=a$

練習 14

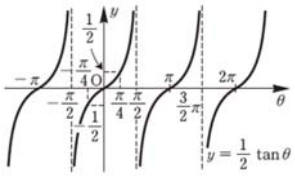
⑴ 周期 2π，値域 $-2\leqq y\leqq 2$



⑵ 周期 2π，値域 $-\dfrac{1}{2}\leqq y\leqq\dfrac{1}{2}$

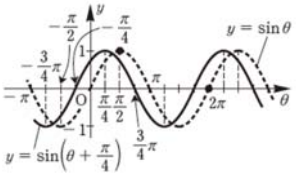


⑶ 周期 π，値域　すべての実数

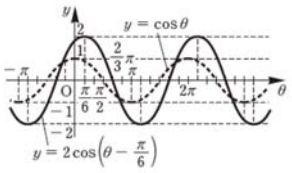


練習 14

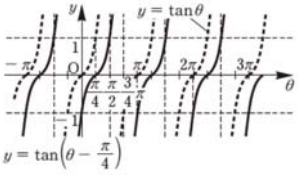
⑴



⑵



⑶



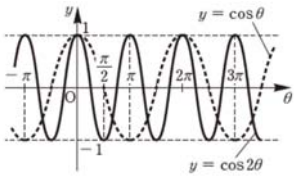
練習 16

$\sin\frac{2}{3}\theta$ のとき 周期を p とすると $p=\frac{2\pi}{\left(\frac{2}{3}\right)}=3\pi$，$y=\sin\frac{2}{3}\theta$ の周期は 3π

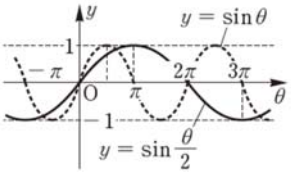
$\tan\frac{2}{3}\theta$ のとき $p=\frac{\pi}{\left(\frac{2}{3}\right)}=\frac{3}{2}\pi$，$\tan\frac{2}{3}\theta$ の周期は $\frac{3}{2}\pi$

練習 17

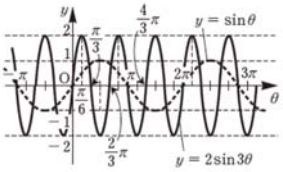
(1) 周期 π



(2) 周期 4π



(3) 周期 $\frac{2}{3}\pi$



練習 18

(1) $\sqrt{2}\sin\theta=1$

$\sin\theta=\frac{1}{\sqrt{2}}$

$\theta=\frac{\pi}{4},\ \frac{3}{4}\pi$

$\theta=\frac{\pi}{4}+2n\pi,\ \frac{3}{4}\pi+2n\pi$

(2) $2\cos\theta+\sqrt{3}=0$

$2\cos\theta=-\sqrt{3}$

$\cos\theta=-\frac{\sqrt{3}}{2}$

$\theta=\frac{5}{6}\pi,\ \frac{7}{6}\pi$

$\theta=\frac{5}{6}\pi+2n\pi,\ \frac{7}{6}\pi+2n\pi$

(3) $\tan\theta=\frac{1}{\sqrt{3}}$

$\theta=\frac{\pi}{6},\ \frac{7}{6}\pi$

$\theta=\frac{\pi}{6}+n\pi$

練習 19

(1) $\sin\theta > \dfrac{\sqrt{3}}{2}$

$\dfrac{\pi}{3} < \theta < \dfrac{2}{3}\pi$

(2) $\cos\theta \leqq \dfrac{1}{2}$

$\dfrac{\pi}{3} \leqq \theta \leqq \dfrac{5}{3}\pi$

(3) $\tan\theta > 1$

$\dfrac{\pi}{4} < \theta < \dfrac{\pi}{2}, \quad \dfrac{5}{4}\pi < \theta < \dfrac{3}{2}\pi$

(4) $\sin\theta \geqq \dfrac{1}{2}$

$0 \leqq \theta \leqq \dfrac{7}{6}\pi, \ \dfrac{11}{6}\pi \leqq \theta < 2\pi$

(5) $-\dfrac{\sqrt{3}}{2} < \cos\theta < \dfrac{1}{2}$

$\dfrac{\pi}{3} < \theta < \dfrac{5}{6}\pi, \ \dfrac{7}{6}\pi < \theta < \dfrac{5}{3}\pi$

練習 20

(1) $\arcsin 1 = x$ とすると $\sin x = 1$

$-\dfrac{\pi}{2} \leqq x \leqq \dfrac{\pi}{2}$ より $x = \dfrac{\pi}{2}$

(2) $\arcsin\left(-\dfrac{1}{2}\right) = x$ とすると $\sin x = -\dfrac{1}{2}$

$-\dfrac{\pi}{2} \leqq x \leqq \dfrac{\pi}{2}$ より $x = -\dfrac{\pi}{6}$

(3) $\arccos 0 = x$ とすると $\cos x = 0$

$0 \leqq x \leqq \pi$ より $x = \dfrac{\pi}{2}$

(4) $\arccos\left(\dfrac{1}{\sqrt{2}}\right) = x$ とすると $\cos x = \dfrac{1}{\sqrt{2}}$

$0 \leqq x \leqq \pi$ より $x = \dfrac{\pi}{4}$

(5) $\arctan\left(-\sqrt{3}\right) = x$ とすると $\tan x = -\sqrt{3}$

$-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$ より $x = -\dfrac{\pi}{3}$

(6) $\arctan(-1) = x$ とすると $\tan x = -1$

$-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$ より $x = -\dfrac{\pi}{4}$

節末問題

1．

⑴　-1　　⑵　$\dfrac{\sqrt{3}}{2}$

⑶　$\sqrt{3}$　　⑷　$\dfrac{\sqrt{3}}{2}$

⑸　-1　　⑹　0

2．

側面の扇形の中心角をθラジアンとする

底面の円周＝側面の扇形の弧の長さより　　$2\pi r = l\theta \quad \therefore \theta = \dfrac{2\pi r}{l}$

このとき面積Sは　　$S = \dfrac{1}{2}l^2\theta = \dfrac{1}{2} \times l^2 \times \dfrac{2\pi r}{l} = \pi l r$

3．

$\sin^2\theta + \cos^2\theta$ より

$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(-\dfrac{1}{3}\right)^2 = \dfrac{8}{9}$

$\pi < \theta < \dfrac{3}{2}\pi$ より $\sin\theta < 0 \quad \therefore \sin\theta = -\dfrac{2\sqrt{2}}{3}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ より $\tan\theta = \dfrac{\left(-\dfrac{2\sqrt{2}}{3}\right)}{\left(-\dfrac{1}{3}\right)} = 2\sqrt{2}$

4．

⑴　$(\sin\theta + \cos\theta)^2 = \left(\dfrac{\sqrt{3}}{2}\right)^2$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = \dfrac{3}{4}$

$\left(\sin^2\theta + \cos^2\theta\right) + 2\sin\theta\cos\theta = \dfrac{3}{4}$

$1 + 2\sin\theta\cos\theta = \dfrac{3}{4}$

$\sin\theta\cos\theta = -\dfrac{1}{8}$

⑵　$\sin^3\theta + \cos^3\theta$

$= (\sin\theta + \cos\theta)^3 - 3\sin\theta\cos\theta(\sin\theta + \cos\theta)$

$= \left(\dfrac{\sqrt{3}}{2}\right)^3 - 3 \times \left(-\dfrac{1}{8}\right) \times \dfrac{\sqrt{3}}{2} = \dfrac{6\sqrt{3} + 3\sqrt{3}}{16} = \dfrac{9\sqrt{3}}{16}$

(3) $\sin\theta - \cos\theta = x$ とする

$(\sin\theta - \cos\theta)^2 = x^2$

$\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta = x^2$

$1 - 2\sin\theta\cos\theta = x^2$

$1 - 2\times\left(-\frac{1}{8}\right) = x^2$

$x^2 = \frac{5}{4}$ より $x = \pm\frac{\sqrt{5}}{2}$

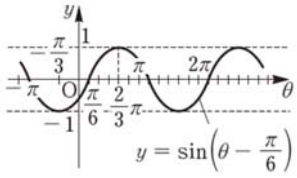
(4) $\pm\frac{7\sqrt{5}}{16}$

5.

$$\cos\left(\theta-\frac{\pi}{2}\right)\sin(\theta+\pi)-\sin\left(\theta+\frac{3}{2}\pi\right)\cos(\theta-\pi)=\sin\theta(-\sin\theta)-(-\cos\theta)(-\cos\theta)$$
$$=-\sin^2\theta-\cos^2\theta=-(\sin^2\theta+\cos^2\theta)=-1$$

6.

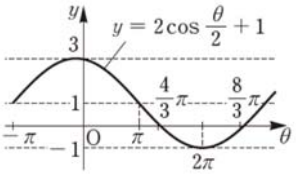
(1)



周期 2π

値域 $-1 \leqq y \leqq 1$

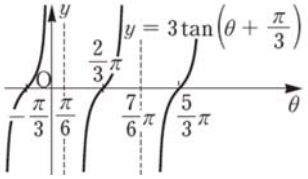
(2)



周期 4π

値域 $-1 \leqq y \leqq 3$

(3)



周期 π

値域 すべての実数

7.

(1) $\theta = \frac{\pi}{3},\ \frac{5}{3}\pi$

(2) $\theta = \frac{4}{3}\pi,\ \frac{5}{3}\pi$

(3) $\theta = \frac{5}{6}\pi,\ \frac{11}{6}\pi$

(4) $0 \leqq \theta \leqq \frac{\pi}{6},\quad \frac{5}{6}\pi \leqq \theta \leqq 2\pi$

(5) $0 \leqq \theta < \frac{\pi}{4},\quad \frac{7}{4}\pi < \theta < 2\pi$

(6) $\frac{\pi}{3} \leqq \theta < \frac{\pi}{2},\quad \frac{4}{3}\pi \leqq \theta < \frac{3}{2}\pi$

8.

(1) $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = x$ とすると $\sin x = -\frac{\sqrt{3}}{2}$

$-\frac{\pi}{2} < x < \frac{\pi}{2}$ より $x = -\frac{\pi}{3}$

(2) $\arccos(-1) = x$ とすると $\cos x = -1$

$0 \leqq x \leqq \pi$ より $x = \pi$

(3) $\arctan\sqrt{3} = x$ とすると $\tan x = \sqrt{3}$

$-\frac{\pi}{2} < x < \frac{\pi}{2}$ より $x = \frac{\pi}{3}$