1−1 三角比と三角関数

問題 1(1) $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$

(2) $\sin\theta = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$, $\cos\theta = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$, $\tan\theta = \frac{3}{3} = 1$

問題 2 $x = 3\cos 60° = 3 \cdot \frac{1}{2} = \frac{3}{2}$ $y = 3\sin 60° = 3 \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

問題 3 $c = \sqrt{3^2 + 5^2} = \sqrt{34}$

問題 4 nは整数とする。

(1) $\theta = 225°$ 一般角 $\theta = 225° + 360° \times n$

(2) $\theta = 270°$ 一般角 $\theta = 270° + 360° \times n$

問題 5

(1) $\sin 45° = \frac{1}{\sqrt{2}}$ $\cos 45° = \frac{1}{\sqrt{2}}$, $\tan 45° = 1$

(2) $\sin 135° = \frac{1}{\sqrt{2}}$ $\cos 135° = -\frac{1}{\sqrt{2}}$, $\tan 135° = -1$

(3) $\sin 150° = \frac{1}{2}$ $\cos 150° = -\frac{\sqrt{3}}{2}$, $\tan 150° = -\frac{1}{\sqrt{3}}$

問題 6 $S = 14 \times 12\sin 30° \div 2 = 14 \times 12\frac{1}{2} \div 2 = 42$

問題 7 余弦定理から $a^2 = 2^2 + 3^2 - 2 \cdot 2 \cdot 3\cos 60° = 7$

よって $a = \sqrt{7}$

問題 8 余弦定理Ⅱから $\cos B = \frac{8^2 + 5^2 - 7^2}{2 \cdot 8 \cdot 5} = \frac{1}{2}$

よって $B = 60°$

練習問題

1 (1) $\sin 210° = -\frac{1}{2}$ $\cos 210° = -\frac{\sqrt{3}}{2}$ $\tan 210° = \frac{1}{\sqrt{3}}$

(2) $\sin 225° = -\frac{1}{\sqrt{2}}$ $\cos 225° = -\frac{1}{\sqrt{2}}$ $\tan 225° = 1$

(3) $\sin 240° = -\frac{\sqrt{3}}{2}$ $\cos 240° = -\frac{1}{2}$ $\tan 240° = \sqrt{3}$

(4) $\sin 300° = -\frac{\sqrt{3}}{2}$ $\cos 300° = \frac{1}{2}$ $\tan 300° = -\sqrt{3}$

(5) $\sin 315° = -\frac{1}{\sqrt{2}}$ $\cos 315° = \frac{1}{\sqrt{2}}$ $\tan 315° = -1$

(6) $\sin 330° = -\frac{1}{2}$ $\cos 330° = \frac{\sqrt{3}}{2}$ $\tan 330° = -\frac{1}{\sqrt{3}}$

2 余弦定理Ⅱから $\cos B = \frac{5^2 + 7^2 - \sqrt{39}^2}{2 \cdot 5 \cdot 7} = \frac{1}{2}$

よって $B = 60°$

1−2 弧度法と三角関数

問題 1　(1) $\sin\frac{\pi}{6}=\frac{1}{2}$,　$\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$,　$\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$

(2) $\sin\frac{5}{6}\pi=\frac{1}{2}$,　$\cos\frac{5}{6}\pi=-\frac{\sqrt{3}}{2}$,　$\tan\frac{5}{6}\pi=-\frac{1}{\sqrt{3}}$

(3) $\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$,　$\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$,　$\tan\frac{\pi}{4}=1$

(4) $\sin\frac{3}{4}\pi=\frac{1}{\sqrt{2}}$,　$\cos\frac{3}{4}\pi=-\frac{1}{\sqrt{2}}$,　$\tan\frac{3}{4}\pi=-1$

練習問題

1　(1)$\frac{\pi}{6}$　(2)$\frac{\pi}{3}$　(3)$\frac{\pi}{2}$　(4)$\frac{2}{3}\pi$　(5)$\frac{5}{6}\pi$　(6)60°　(7)180°　(8)240°　(9) 315°　(10)360°

2　(1) $\sin\frac{5}{4}\pi=-\frac{1}{\sqrt{2}}$　$\cos\frac{5}{4}\pi=-\frac{1}{\sqrt{2}}$　$\tan\frac{5}{4}\pi=1$

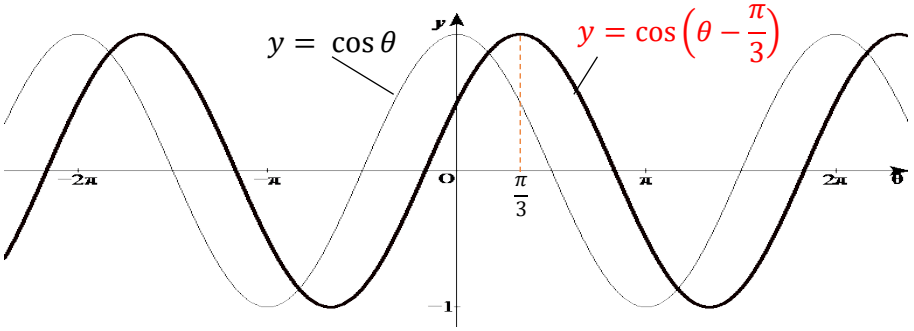
(2) $\sin\frac{11}{6}\pi=-\frac{1}{2}$　$\cos\frac{11}{6}\pi=\frac{\sqrt{3}}{2}$　$\tan\frac{11}{6}\pi=-\frac{1}{\sqrt{3}}$

(3) $\sin\pi=0$　$\cos\pi=-1$　$\tan\pi=0$

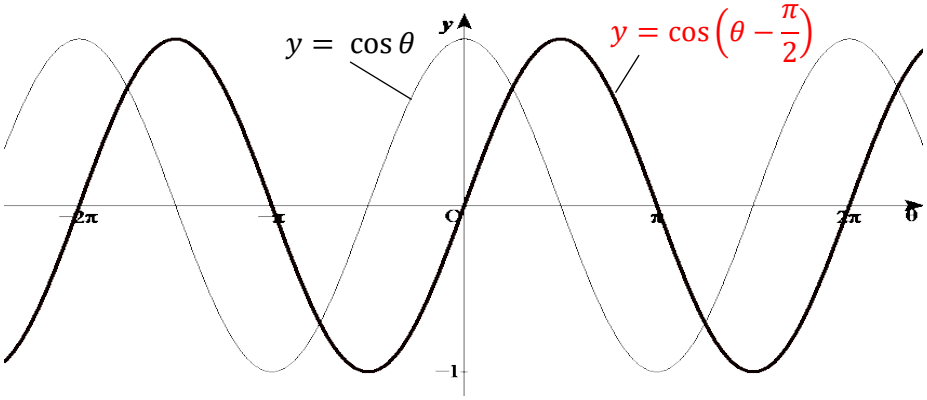
(4) $\sin\frac{\pi}{2}=1$　$\cos\frac{\pi}{2}=0$　$\tan\frac{\pi}{2}$は定義されない

1−3 三角関数のグラフ

問題 1(1) $y=\cos\left(\theta-\frac{\pi}{3}\right)$のグラフは$y=\cos\theta$のグラフを$\theta$軸方向に$\frac{\pi}{3}$平行移動したもの



(2) $y=\cos\left(\theta-\frac{\pi}{2}\right)$のグラフは$y=\cos\theta$のグラフを$\theta$軸方向に$\frac{\pi}{2}$平行移動したもの

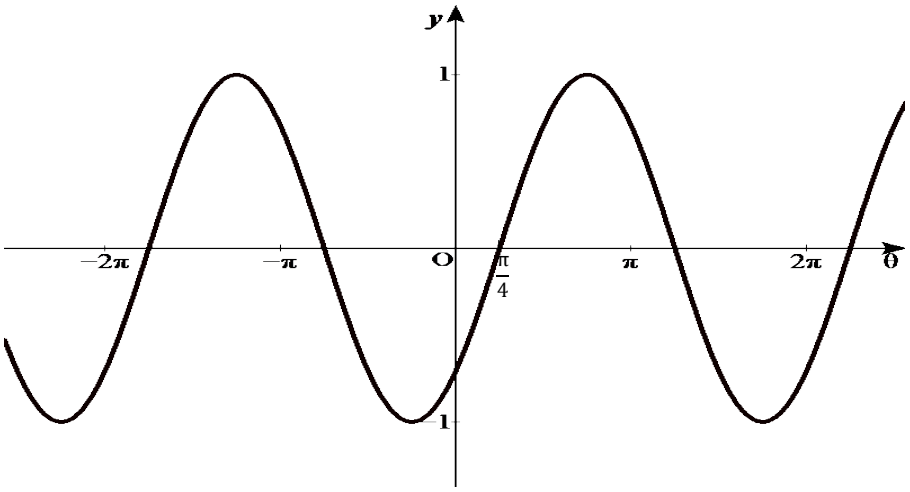


問題 2 (1)$\frac{\pi}{2}$　(2)8π　(3) T

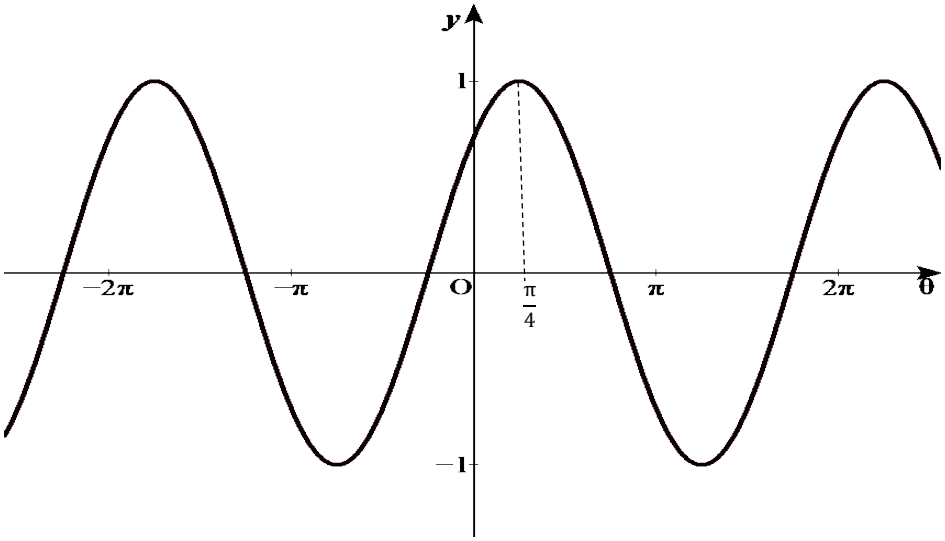
問題 3 最大値は 3、最小値は－3

練習問題

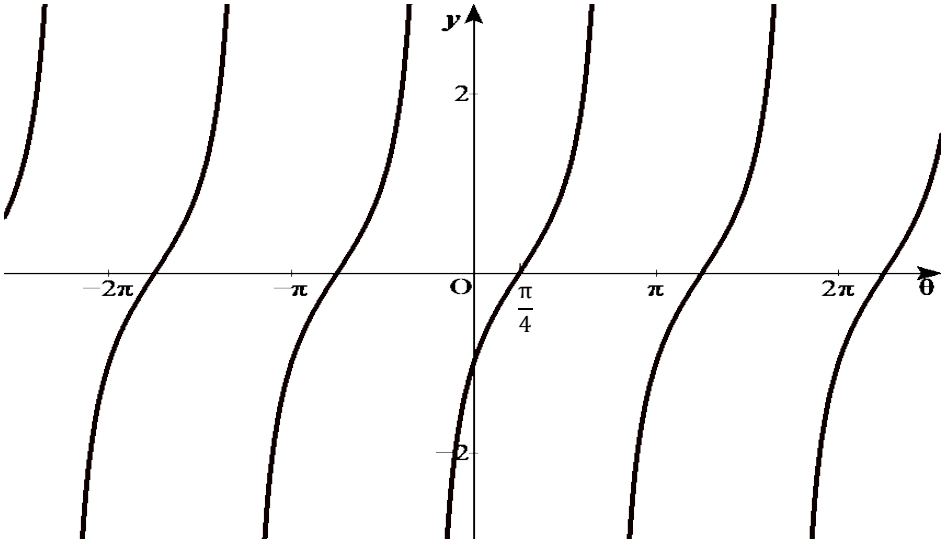
1(1)



(2)

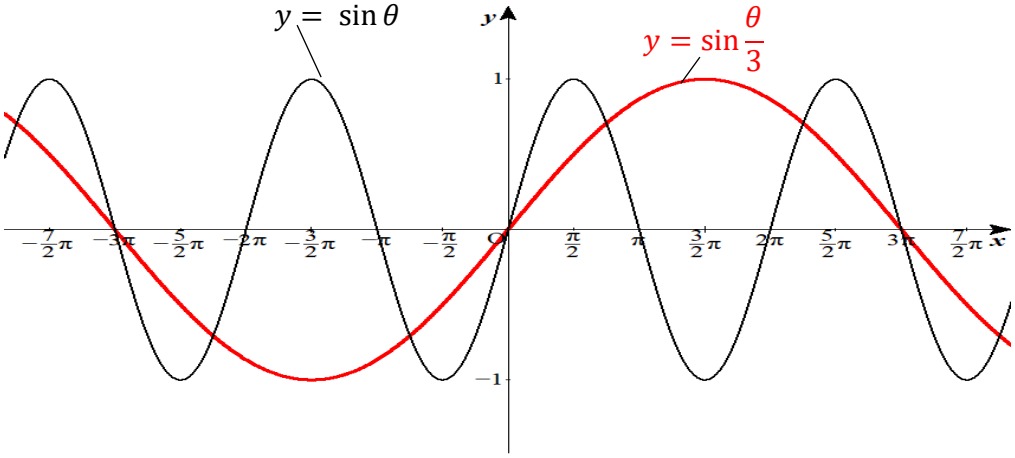


(3)



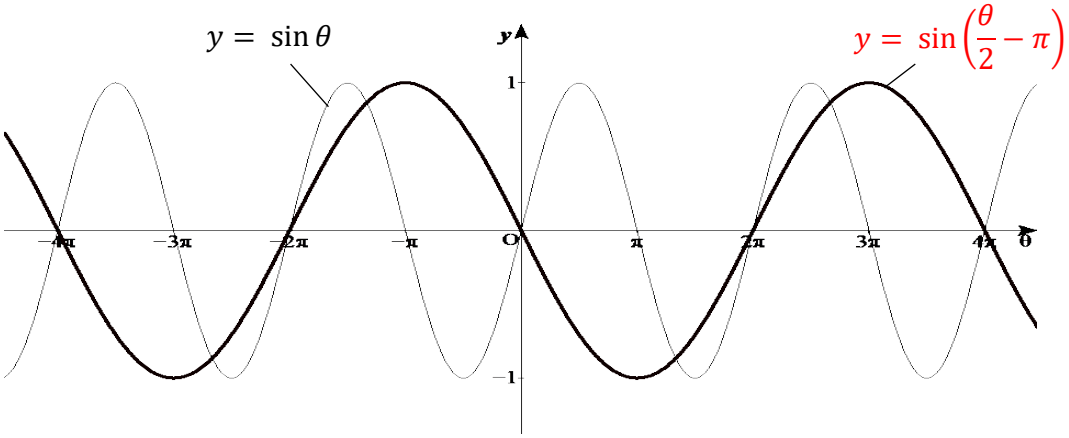
問題 2

(1) $y=\sin\frac{\theta}{3}$のグラフは$y=\sin\theta$のグラフをθ軸方向に 3 倍したもの



(2) $y=\sin\left(\frac{\theta}{2}-\pi\right)=\sin\left(\frac{1}{2}(\theta-2\pi)\right)$ のグラフは、$y=\sin\theta$ のグラフをθ軸方向に 2 倍

した $y=\sin\frac{\theta}{2}$ のグラフをθ軸方向に 2πだけ平行移動したもの



問題 3(1)最大値 5　　最小値－5

(2)最大値 5　　最小値－5

(3)最大値 3　　最小値－3

(4)最大値 3　　最小値－1

(5)最大値 5　　最小値－5

1－4 三角関数の方程式・不等式

問題 1(1)$\theta=\frac{5}{6}\pi,\frac{7}{6}\pi$　(2)一般角では、$\theta=\frac{5}{6}\pi+2n\pi,\quad\frac{7}{6}\pi+2n\pi$ (nは整数)

問題 2 (1)$\frac{5}{6}\pi,\frac{11}{6}\pi$　　(2)一般角では、$\theta=\frac{5}{6}\pi+n\pi$ (nは整数)

問題 3　$\sin^2\theta=1-\cos^2\theta$ を与式に代入して整理すると $2\cos^2\theta-\cos\theta=0$を得る

$$\cos\theta=t\quad\text{とおくと}\quad 2t^2-t=0$$

$$t(2t-1)=0\quad\text{より}\qquad t=0,\frac{1}{2}$$

$\cos\theta=0$ のとき　$\theta=\frac{\pi}{2},\frac{3}{2}\pi$

$\cos\theta = \frac{1}{2}$のとき　$\theta = \frac{\pi}{3}, \frac{5}{3}\pi$

問題 4 (1) $\frac{\pi}{4} \leq \theta \leq \frac{7}{4}\pi$　(2) $0 \leq \theta < \frac{\pi}{4}$,　　$\frac{\pi}{2} < \theta < \frac{5}{4}\pi$,　$\frac{3}{2}\pi < \theta < 2\pi$

1−5 加法定理

問題 1 $\sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3} = \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}$

問題 2 まず、問題 1 と同様に$\cos\frac{7\pi}{12}$を求めると

$\cos\frac{7\pi}{12} = \frac{\sqrt{2}-\sqrt{6}}{4}$

よって

$\tan\frac{7\pi}{12} = \frac{\sin\frac{7\pi}{12}}{\cos\frac{7\pi}{12}} = \frac{\frac{\sqrt{2}+\sqrt{6}}{4}}{\frac{\sqrt{2}-\sqrt{6}}{4}} = \frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}-\sqrt{6}} = -2-\sqrt{3}$

問題 3 $\frac{\pi}{2} \leq \alpha \leq \pi$ では　$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\left(-\frac{1}{3}\right)^2} = \frac{2\sqrt{2}}{3}$

よって

(1) $\sin 2\alpha = 2\sin\alpha\cos\alpha = 2 \cdot \frac{2\sqrt{2}}{3} \cdot \frac{-1}{3} = \frac{-4\sqrt{2}}{9}$

(2) $\cos 2\alpha = 2\cos^2\alpha - 1 = 2 \cdot \left(-\frac{1}{3}\right)^2 - 1 = -\frac{7}{9}$

問題 4

(1) $\sin^2\frac{\pi}{8} = \frac{1-\cos\frac{\pi}{4}}{2} = \frac{1-\frac{1}{\sqrt{2}}}{2} = \frac{2-\sqrt{2}}{4}$

$\sin\frac{\pi}{8} > 0$ より　$\sin\frac{\pi}{8} = \sqrt{\frac{2-\sqrt{2}}{4}}$

(2) $\cos^2\frac{\pi}{8} = \frac{1+\cos\frac{\pi}{4}}{2} = \frac{1+\frac{1}{\sqrt{2}}}{2} = \frac{2+\sqrt{2}}{4}$

$\cos\frac{\pi}{8} > 0$ より　$\cos\frac{\pi}{8} = \sqrt{\frac{2+\sqrt{2}}{4}}$

問題 5　(1) $\sqrt{3}\sin\theta + \cos\theta = 2\left(\sin\theta \cdot \frac{\sqrt{3}}{2} + \cos\theta \cdot \frac{1}{2}\right) = 2\left(\sin\theta \cdot \cos\frac{\pi}{6} + \cos\theta \cdot \sin\frac{\pi}{6}\right)$

$= 2\sin\left(\theta + \frac{\pi}{6}\right)$

(2) $\frac{5}{\sqrt{2}}\sin\theta + \frac{5}{\sqrt{2}}\cos\theta = 5\left(\sin\theta \cdot \frac{1}{\sqrt{2}} + \cos\theta \cdot \frac{1}{\sqrt{2}}\right) = 5\left(\sin\theta \cdot \cos\frac{\pi}{4} + \cos\theta \cdot \sin\frac{\pi}{4}\right) = 5\sin\left(\theta + \frac{\pi}{4}\right)$

問題 6. $\sqrt{3}\sin\theta+\cos\theta=2\sin\left(\theta+\frac{\pi}{6}\right)$　より、最大値は 2、最小値は -2

練習問題

1　(1) $\sin\theta-\cos\theta=\sqrt{2}\left(\sin\theta\cdot\frac{1}{\sqrt{2}}+\cos\theta\cdot\left(-\frac{1}{\sqrt{2}}\right)\right)=\sqrt{2}\left(\sin\theta\cdot\cos\frac{7}{4}\pi+\cos\theta\cdot\sin\frac{7}{4}\pi\right)$

$$=\sqrt{2}\sin\left(\theta+\frac{7}{4}\pi\right)$$

(2) $\sqrt{3}\sin\theta-\cos\theta=2\left(\sin\theta\cdot\frac{\sqrt{3}}{2}+\cos\theta\cdot\left(-\frac{1}{2}\right)\right)=2\left(\sin\theta\cdot\cos\frac{11}{6}\pi+\cos\theta\cdot\sin\frac{11}{6}\pi\right)$

$$=2\sin\left(\theta+\frac{11}{6}\pi\right)$$

1−6 極座標

問題 1　$r=\sqrt{1^2+\left(-\sqrt{3}\right)^2}=2$

$\cos\theta=\frac{1}{2}$,　$\sin\theta=\frac{-\sqrt{3}}{2}$　　$0\leq\theta<2\pi$より　$\theta=\frac{7}{4}\pi$

よって　　$\left(2,\frac{7}{4}\pi\right)$

問題 2　$x=8\cos\frac{2}{3}\pi=-4$,　$y=8\sin\frac{2}{3}\pi=4\sqrt{3}$

より$\left(-4\,,4\sqrt{3}\right)$

問題 3

$x=r\sin\theta\cos\varphi=5\sin\frac{\pi}{4}\cos\frac{\pi}{3}=2$

$y=r\sin\theta\sin\varphi=5\sin\frac{\pi}{4}\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$

$z=r\cos\theta=2\cos\frac{\pi}{4}=\sqrt{2}$

よって

$$\left(\frac{5\sqrt{2}}{4}\,,\frac{5\sqrt{6}}{4},\sqrt{2}\right)$$