

## 2. 指数関数と対数関数

### ■1■ べき乗 : P30

問題1. (1)  $2^0=1$

$$(3) 0.5^{-2} = \left(-\frac{1}{2}\right)^{-2} = (2^{-1})^{-2} = 2^2 = 4$$

$$(2) 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

$$(4) 10^{-5} = \frac{1}{10^5} = \frac{1}{100000} = 0.00001$$

問題2. (1)  $81^{\frac{3}{4}} = (3^4)^{\frac{3}{4}} = 3^3 = 27$

$$(2) 2^{-\frac{3}{2}} = (\sqrt{2})^3 = 2\sqrt{2}$$

$$(3) 4^{-\frac{1}{2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

$$(4) 25^{-\frac{3}{2}} = (5^2)^{-\frac{3}{2}} = 5^{-3} = \frac{1}{5^3} = \frac{1}{125}$$

$$(5) \sqrt[3]{9 \cdot \sqrt[3]{81}} = (3^2)^{\frac{1}{3}} \cdot (3^4)^{\frac{1}{3}} = 3^{-\frac{2}{3}} \cdot 3^{\frac{4}{3}} = 3^{\frac{2}{3}} \cdot 3^{\frac{4}{3}} = 3^2 = 9$$

$$(6) \sqrt[6]{4096} = \{(2^{12})^{\frac{1}{2}}\}^{\frac{1}{6}} = 2^1 = 2$$

### ■2■ 指数法則 : P31

問題1. (1)  $(-2)^3 \times 2^2 = -8 \times 4 = -32$

$$(2) \{(2^{-1})^{-3}\}^2 = (2^3)^2 = 2^6 = 64$$

$$(2) \{(-3)^2\}^2 = (-3)^4 = 81$$

$$(4) 7^4 \div 7^2 = 7^4 \times 7^{-2} = 7^2 = 49$$

問題2. (1)  $\left(\frac{1}{8}\right)^{\frac{1}{2}} \times \frac{1}{8} = (2^{-3})^{\frac{1}{2}} \times 2^{-3} = 2^{-\frac{3}{2}} \times 2^{-3} = 2^{-\frac{9}{2}} = \frac{1}{\sqrt{2}^9} = \frac{1}{16\sqrt{2}}$

$$(2) \left(\frac{1}{9}\right)^2 \times 3^{-\frac{9}{2}} = (3^{-2})^2 \times 3^{-\frac{9}{2}} = 3^{-4} \times 3^{-\frac{9}{2}} = 3^{-\frac{17}{2}} = \sqrt{3}$$

$$(3) 8^{\frac{1}{4}} \div 2^{-\frac{3}{4}} = (2^3)^{\frac{1}{4}} \times 2^{\frac{3}{4}} = 2^{\frac{3}{4}} \times 2^{\frac{3}{4}} = 2^{\frac{3}{2}} = 2\sqrt{2}$$

$$(4) (2 \times 3^{\frac{1}{3}})^6 = 2^6 \times (3^{\frac{1}{3}})^6 = 2^6 \times 3^2 = 64 \times 9 = 576$$

問題3. (1)  $(x^{\frac{5}{3}} \cdot y^{\frac{2}{4}})^{12} \cdot (xy)^2 = x^{20} \cdot y^9 \cdot x^2 \cdot y^2 = x^{22} \cdot y^{11}$

$$(2) (x^4 \cdot y^6)^{\frac{1}{2}} = (x^4)^{\frac{1}{2}} \cdot (y^6)^{\frac{1}{2}} = x^2 \cdot y^3$$

$$(3) x^2 \div y^3 \times y^{\frac{1}{2}} = x^2 \times y^{-3} \times y^{\frac{1}{2}} = x^2 \cdot y^{-\frac{5}{2}}$$

### ■3■ 指数関数のグラフ : P32

問題1. 略

問題2. 略

問題3. (1)  $\sqrt{3} = 3^{\frac{1}{2}}$ ,  $\sqrt[3]{9} = 3^{\frac{2}{3}}$ ,  $\sqrt[4]{27} = 3^{\frac{3}{4}}$

$$\frac{1}{2} < \frac{2}{3} < \frac{3}{4} \text{ より、}$$

$$\sqrt{3} < \sqrt[3]{9} < \sqrt[4]{27}$$

$$(2) \sqrt{\frac{1}{2}} = 2^{-\frac{1}{2}}, \sqrt[3]{\frac{1}{4}} = 2^{-\frac{2}{3}}, \sqrt[4]{\frac{1}{8}} = 2^{-\frac{3}{4}}$$

$$-\frac{3}{4} < -\frac{2}{3} < -\frac{1}{2} \text{ より、}$$

$$\sqrt[4]{\frac{1}{8}} < \sqrt[3]{\frac{1}{4}} < \sqrt{\frac{1}{2}}$$

練習1. 略

### ■4■ 対数とその性質 : P34

問題1. (1)  $\log_2 256 = 8$  (2)  $\log_{10} \frac{1}{10} = -1$  (3)  $\log_2 \frac{\sqrt{2}}{2} = -\frac{1}{2}$  (4)  $\log_3 \sqrt{27} = \frac{3}{2}$

問題2. (1)  $4^3 = 64$  (2)  $3^{-2} = \frac{1}{9}$  (3)  $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$  (4)  $\left(\frac{1}{2}\right)^{-3} = 8$

問題3. (1)  $\log_2 4\sqrt{2} = \log_2 2^{\frac{5}{2}} = \frac{5}{2}$  (2)  $\log_8 2 = \log_8 8^{-\frac{1}{3}} = \frac{1}{3}$  (3)  $\log_8 \frac{1}{64} = \log_2 2^{-6} = -6$

$$(4) \log_2 \sqrt[6]{64} = \log_2 (2^6)^{\frac{1}{6}} = \log_2 2 = 1$$

問題4. (1)  $\log_{10} 2 + \log_{10} 5 = \log_{10} (2 \times 5) = \log_{10} 10 = 1$

$$(2) \log_2 \sqrt{8} + \log_2 \sqrt{32} = \log_2 (2^{\frac{3}{2}} \times 2^{\frac{5}{2}}) = \log_2 2^4 = 4$$

$$(3) \log_9 27 - \log_9 3 = \log_9 \frac{27}{3} = \log_9 9 = 1$$

$$(4) \log_5 15 - \log_5 75 = \log_5 \frac{15}{75} = \log_2 \frac{1}{5} = \log_5 5^{-1} = -1$$

問題5. (1)  $\log_{0.5}4 + \log_4 0.25 = \log_2 2 = \log_{2^{-1}} 2^2 + \log_2 2^{-2} = -2\log_2 2 - 2\log_2 2 = -2 - 2 = -4$

(2)  $\log_2 5 + \log_4 0.04 = \log_2 5 + \log_{2^2} 5^{-2} = \log_2 5 - \frac{2}{2} \log_2 5 = \log_2 5 - \log_2 5 = 0$

(3)  $\log_2 3 \times \log_3 8 = \log_2 3 \times \frac{\log_2 2^3}{\log_2 3} = 3$

(4)  $\log_5 9 \times \log_3 5 = \log_5 3^2 \times \frac{1}{\log_5 3} = 2$

(5)  $\log_3 5 \times \log_5 2 \times \log_2 3 = \frac{\log_2 5}{\log_2 3} \times \frac{1}{\log_2 5} \times \log_2 3 = 1$

練習1. (1)  $\log_2 \sqrt[3]{2} = \log_2 2^{\frac{1}{3}} = \frac{1}{3}$

(2)  $\log_{10} \frac{1}{100000} = \log_{10} 10^{-5} = -5$

(3)  $\log_{0.5} 4 = \log_{2^{-1}} 2^2 = -2$

(4)  $\log_7 7\sqrt{7} = \log_7 7^{\frac{3}{2}} = \frac{3}{2}$

(5)  $\log_3 \sqrt[3]{12} - \frac{2}{3} \log_3 2 = \log_2 (3 \times 2^2)^{\frac{1}{3}} - \frac{2}{3} \log_3 2 = \log_3 3^{\frac{1}{3}} + \log_3 2^{\frac{2}{3}} - \frac{2}{3} \log_3 2 = \frac{1}{3}$

(6)  $\frac{2}{3} \log_2 3 + \log_2 \sqrt[3]{3} = \log_2 3^{\frac{2}{3}} + \log_2 2 - \log_2 3^{-\frac{1}{2}} = \log_2 (3^{\frac{2}{3}} \times 3^{-\frac{1}{2}}) + 1 = \log_2 3^{\frac{1}{6}} + 1 = \log_2 \sqrt[6]{3} + 1$

(7)  $\log_7 25 \times \log_5 49 = \log_7 5^2 \times \log_5 7^2 = 2\log_7 5 \times \frac{2}{\log_7 5} = 4$

(8)  $\log_2 5 \times \log_5 7 \times \log_2 8 = \log_2 5 \times \frac{\log_2 7}{\log_2 5} \times \log_2 2^3 = 3\log_2 7$

■5■ 対数関数のグラフ : P36

問題1. 略

問題2. 略

問題3. (1)  $\log_2 5, \log_4 9 = \log_2 3, \log_8 17 = \frac{1}{3} \log_2 17 = \log_2 17^{\frac{1}{3}}$  (2)  $\log_{\frac{1}{2}} \frac{1}{3} = \log_2 3, \log_{\frac{1}{4}} \frac{1}{6} = \log_2 6^{\frac{1}{2}}, \log_{\frac{1}{8}} \frac{1}{10} = \log_2 10^{\frac{1}{3}}$   
 $5^3 = 125, 3^3 = 27, (17^{\frac{1}{3}})^3 = 17$  より、  
 $\log_8 17 < \log_4 9 < \log_2 5$        $3^6 = 729, (6^{\frac{1}{2}})^6 = 216, (10^{\frac{1}{3}})^6 = 100$  より、  
 $\log_{\frac{1}{8}} \frac{1}{10} < \log_{\frac{1}{4}} \frac{1}{6} < \log_{\frac{1}{2}} \frac{1}{3}$

練習1. 略