3. 複素数

■1■ 複素数の計算 : P114

問題1.

(1) $-2x+(3x-5y)i=3+3i$

$-2x=3 \quad \therefore x=-\frac{3}{2}$

$3x-5y=3 \quad y=-\frac{3}{2}$

(2) $(3y-4x)+(3x-2y)i=1$

$3y-4x=1 \quad \therefore x=2$

$3x-2y=0 \quad y=3$

(3) $5x-2y+4yi=6+8i$

$5x-2y=6 \quad \therefore x=2$

$4y=8 \quad y=2$

(4) $\frac{4x+3yi}{5}=-6+2i$

$\frac{4}{5}x=-6 \quad \therefore x=-\frac{15}{2}$

$\frac{3}{5}y=2 \quad y=\frac{10}{3}$

問題2.

(1) $(2+8i)+(9-6i)$

$=(2+9)+(8-6)i$

$=11+2i$

(2) $(-9-7i)-(-8i)$

$=-9+(-7+8)i$

$=-9+i$

(3) $(-5+3i)^2=25-30i-9$

$=16-30i$

(4) $(-9-6i)\div(-2i)=\frac{-9}{-2i}+\frac{-6i}{-2i}$

$=3-\frac{9}{2}i$

(5) $(1-2i)\div(3+4i)$

$=\frac{1-2i}{3+4i}=\frac{(1-2i)(3-4i)}{(3+4i)(3-4i)}$

$=\frac{-5-10i}{25}=\frac{-1-2i}{5}$

問題3.

(1) $z_1+z_2=(2+3i)+(-3+2i)$

$=-1+5i$

(2) $z_1-z_2=(2+3i)-(-3+2i)$

$=5+i$

(3) $z_3\times z_4=(-2-3i)(3-2i)$

$=-6-9i+4i-6$

$=-12-5i$

(4) $z_1\div z_4=\frac{2+3i}{3-2i}=\frac{(2+3i)(3+2i)}{(3-2i)(3+2i)}$

$=\frac{13i}{13}=i$

(5) $(z_4)^2=(3-2i)^2=9-12i-4$

$=5-12i$

(6) $z_2\times z_3+z_1$

$=(-3+2i)(-2-3i)+2+3i$

$=(12+5i)+(2+3i)$

$=14+8i$

(7) $z_2+z_4\times z_2=z_2(1+z_4)$

$=(-3+2i)(1+3-2i)$

$=(-3+2i)(4-2i)=-8+14i$

(8) $z_1-z_2\div z_4=(2+3i)-\frac{-3+2i}{3-2i}$

$=(2+3i)+\frac{3-2i}{3-2i}=2+3i+1$

$=3+3i$

■2■ 方程式と複素数 : P116

問題1.

(1) $4x^2=-25$

$x^2=-\frac{25}{4}$

$x=\pm\frac{5}{2}i$

(2) $3x^2+8=0$

$x^2=-\frac{8}{3}$

$x=\pm\frac{2\sqrt{2}}{\sqrt{3}}i$

問題2.

(1) $x^2+x+1=0$

$x=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm\sqrt{3}i}{2}$

(2) $x^2-4x+6=0$

$x=\frac{4\pm\sqrt{16-24}}{2}=2\pm\sqrt{2}i$

問題3.

(1) $x^4-16=0$

$(x-2)(x+2)(x^2-4)=0$

$x=\pm2,\ \pm2i$

(2) $x^3+2x^2+x+2=0$

$(x+2)(x^2+1)=0$

$x=-2,\ \pm i$

(3) $x^4+2x^2+1=0$

$(x^2+1)^2=0$

$x=\pm i$

練習1.

(1) $x^2-2x+2=0$

$x=\frac{2\pm\sqrt{4-8}}{2}=1\pm i$

(2) $x^3+6x^2+12x=0$

$x(x^2+6x+12)=0$

$x=0,\ -3\pm\sqrt{3}i$

(3) $x^4+10x^2+25=0$

$(x^2+5)^2=0$

$x=\pm\sqrt{5}i$

■3■ 複素平面と指数関数形式 : P118

問題1. 略

問題2. 図は略

(1) $\sqrt{4^2+4^2}e^{i\tan^{-1}\frac{4}{4}}$

$=4\sqrt{2}e^{i\frac{\pi}{4}}$

(2) $3e^{i\frac{\pi}{2}}$

(3) $4\cos\frac{3}{4}\pi+i\,4\sin\frac{3}{4}\pi$

$=-2\sqrt{2}+2\sqrt{2}i$

(4) $5\cos(-\tan^{-1}\frac{4}{3})+i\,5\sin(-\tan^{-1}\frac{4}{3})$

$=4-3i$

問題3. (1) $e^{i\frac{3}{4}\pi}$ (2) $3e^{i\frac{2}{5}\pi}$ (3) $7e^{-i\pi}$ (4) $e^{i\tan^{-1}\frac{3}{7}}$

問題4.

(1) 与式$=3\times4e^{i(\frac{2}{7}\pi+\frac{7}{8}\pi)}=12e^{i\frac{65}{56}\pi}$

(2) 与式$=5e^{i\frac{3}{4}\pi}\times\frac{1}{2}e^{i\frac{3}{2}\pi}=\frac{5}{2}e^{i\frac{9}{4}\pi}=\frac{5}{2}e^{i\frac{1}{4}\pi}$

問題5. (1) $z_3 \times z_4 = e^{-i\frac{5}{6}\pi} \times 2e^{i\frac{7}{6}\pi} = 2e^{i\frac{1}{3}\pi}$

(2) $z_1 \times z_5 = e^{i\frac{1}{3}\pi} \times 2e^{-i\frac{3}{2}\pi} = 2e^{-i\frac{7}{6}\pi}$

(3) $z_5 \div z_3 = 2e^{-i\frac{3}{2}\pi} \div e^{-i\frac{5}{6}\pi} = 2e^{-i\frac{3}{2}\pi} \times e^{i\frac{5}{6}\pi} = 2e^{-i\frac{2}{3}\pi}$

(4) $z_4 \div z_2 = 2e^{i\frac{7}{6}\pi} \div e^{i\pi} = 2e^{i\frac{7}{6}\pi} \times e^{-i\pi} = 2e^{i\frac{1}{6}\pi}$

(5) $z_1 \times z_2 \times z_3 = e^{i\frac{1}{3}\pi} \times e^{i\pi} \times e^{-i\frac{5}{6}\pi} = 2e^{i\frac{1}{2}\pi}$

(6) $z_5 \div z_1 \times z_3 = e^{-i\frac{3}{2}\pi} \div e^{i\frac{1}{3}\pi} \times e^{-i\frac{5}{6}\pi}$

$= e^{-i\frac{3}{2}\pi} \times e^{-i\frac{1}{3}\pi} \times e^{-i\frac{5}{6}\pi} = 2e^{-i\frac{8}{3}\pi} = 2e^{-i\frac{2}{3}\pi}$

■3■ 複素平面と指数関数形式：P118

練習1. 略

練習2. 図は略

(1) $\sqrt{2^2+2^2}\,e^{i\tan^{-1}\frac{2}{2}}$
$=2\sqrt{2}\,e^{i\frac{\pi}{4}}$

(2) $4\cos(-\frac{1}{3}\pi)+i\,4\sin(-\frac{1}{3}\pi)$
$=2-2\sqrt{3}\,i$

(3) $3\cos(-\frac{2}{3}\pi)+i\,3\sin(-\frac{2}{3}\pi)$
$=-\frac{3}{2}-\frac{3\sqrt{3}}{2}\,i$

練習3. 図は略

(1) $\sqrt{(-3)^2+3^2}\,e^{i\tan^{-1}\frac{3}{-3}}$
$=3\sqrt{2}\,e^{i\frac{3}{4}\pi}$

(2) $e^{-i\frac{1}{2}\pi}$

(3) $6\cos(-\frac{1}{4}\pi)+i\,6\sin(-\frac{1}{4}\pi)$
$=3\sqrt{2}-3\sqrt{2}\,i$

(4) $10e^{i\tan^{-1}\frac{3}{4}}=10\cos(\tan^{-1}\frac{3}{4})+i\,10\sin(\tan^{-1}\frac{3}{4})=10\times\frac{4}{5}+10\times\frac{3}{5}i=8+6i$

練習4. (1) $e^{-i\frac{1}{5}\pi}$ (2) $21e^{i\frac{1}{4}\pi}$ (3) $4e^{i\pi}$ (4) $2e^{-i\tan^{-1}\frac{1}{4}}$

練習5. (1) $\frac{1}{2}e^{-i\frac{2}{5}\pi} \times \frac{1}{2}e^{-i\frac{3}{4}\pi} = \frac{1}{4}e^{-i\frac{23}{20}\pi}$

(2) $\frac{1}{3}e^{i\frac{5}{6}\pi} \div \frac{1}{5}e^{i\frac{3}{4}\pi} = \frac{1}{3}e^{i\frac{5}{6}\pi} \times 5e^{-i\frac{3}{4}\pi} = \frac{5}{3}e^{i\frac{1}{12}\pi}$

練習6. (1) $z \times i = 2e^{i\frac{1}{3}\pi} \times e^{i\frac{1}{2}\pi} = 2e^{i\frac{5}{6}\pi}$

(2) $z \times i^2 = 2e^{i\frac{1}{3}\pi} \times (e^{i\frac{1}{2}\pi})^2 = 2e^{i\frac{4}{3}\pi}$

(3) $z \times i^3 = 2e^{i\frac{1}{3}\pi} \times (e^{i\frac{1}{2}\pi})^3 = 2e^{i\frac{11}{6}\pi}$

(4) $z \div i = 2e^{i\frac{1}{3}\pi} \div e^{i\frac{1}{2}\pi} = 2e^{i\frac{1}{3}\pi} \times 2e^{-i\frac{1}{2}\pi} = 2e^{-i\frac{1}{6}\pi}$

■4■ 応用問題：P122

問題1. (1) 与式$=(e^{-i\frac{1}{3}\pi})^6=e^{-i2\pi}=1$ (1) 与式$=(2e^{i\frac{1}{3}\pi})^5=32e^{i\frac{5}{3}\pi}$

問題2. 略

問題3. (1) $V=\frac{8}{\sqrt{2}}e^{j0}=4\sqrt{2}$

(2) $V=\frac{1}{5}e^{j\frac{1}{6}\pi}$

(3) $v(t)=\sqrt{6}\times\sqrt{2}\sin(\omega t+\frac{\pi}{7})$
$=2\sqrt{3}\sin(\omega t+\frac{\pi}{7})$

(4) $i(t)=7\sqrt{2}\times\sqrt{2}\sin(\omega t-\frac{\pi}{9})$
$=14\sin(\omega t-\frac{\pi}{9})$

問題4. (1) $R+j\omega L=5+j100\cdot 0.05$
$=5+j5$ [Ω]

(2) $R-j\frac{1}{\omega C}=5-j\frac{1}{100\cdot 200\times 10^{-6}}$
$=5-j50$ [Ω]

(3) $R+j(\omega L-\frac{1}{\omega C})=5+j(5-50)$
$=5-j45$ [Ω]

問題5. (1) $Z=R+j\omega L$
$=10+j10=10\sqrt{2}\,e^{j\frac{1}{4}\pi}$

$V=50\sqrt{2}$

$I=\frac{V}{Z}=5e^{-j\frac{1}{4}\pi}$

$i(t)=5\sqrt{2}\sin(50t-\frac{1}{4}\pi)$

(2) $Z=R-j\frac{1}{\omega C}$
$=5-j5=5\sqrt{2}\,e^{-j\frac{1}{4}\pi}$

$V=25\sqrt{2}$

$I=\frac{V}{Z}=5e^{j\frac{1}{4}\pi}$

$i(t)=5\sqrt{2}\sin(200t+\frac{1}{4}\pi)$

(3) $Z=R+j(\omega L-\frac{1}{\omega C})$
$=6+j8=5\ 2\,e^{j\frac{1}{4}\pi}$

$V=10$

$I=\frac{V}{Z}=e^{-i\tan^{-1}\frac{4}{3}}$

$i(t)=\sqrt{2}\sin(1000t-\tan^{-1}\frac{4}{3})$