第３章　ワークシート解答

１．

(1)　$e(\omega t) = \begin{cases} E_{\mathrm{m}} \sin \omega t & (0 \leqq \omega t < \pi) \\ 0 & (\pi \leqq \omega t < 2\pi) \end{cases} \qquad E_{\mathrm{m}} = 20$

$$\boldsymbol{V}_0 = \frac{1}{T}\int_0^T e(\omega t)\,d\,\omega t = \frac{1}{2\pi}\int_0^{\pi} E_{\mathrm{m}} \sin \omega t\, d\omega t = \frac{1}{2\pi} 2E_{\mathrm{m}} = \frac{20}{\pi}$$

（答）　6.37 V

$$\boldsymbol{V}_{\mathrm{a}} = \sqrt{\frac{1}{T}\int_0^T \{e(\omega t)\}^2\, d\,\omega t} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} E_{\mathrm{m}}{}^2 \sin^2 \omega t\, d\omega t} = \frac{E_{\mathrm{m}}}{2} = 10$$

（答）　10 V

(2)　$\boldsymbol{V}_0 = \frac{1}{T}\int_0^T \left| e(t)dt \right| = \frac{1}{20\times 10^{-3}}\left\{\int_0^{5\times10^{-3}} 80\,dt + \int_{5\times10^{-3}}^{20\times10^{-3}} 0\,dt\right\} = 20$　　（答）　20 V

$$\boldsymbol{V}_{\mathrm{a}} = \sqrt{\frac{1}{T}\int_0^T \{e(t)\}^2\, dt} = \sqrt{\frac{1}{20\times10^{-3}}\int_0^{5\times10^{-3}} 6400\,dt} = 40$$

（答）　40 V

２．　$v(t) = t^2 \qquad (-1 \leqq t \leqq t) \qquad T = 2$

$$\boldsymbol{V}_0 = \frac{1}{T}\int_0^T v(t)dt = \frac{2}{2}\int_0^1 t^2\,dt = \left[\frac{1}{3}t^3\right]_0^1 = \frac{1}{3}$$

$$\begin{aligned}
\boldsymbol{V}_{\mathrm{A}n} &= \frac{2}{T}\int_0^T v(t)\cos n\omega t\,dt \\
&= \frac{2}{2}\times 2\int_0^1 t^2 \cos n\omega t\,dt \\
&= 2\left\{\left[t^2\frac{\sin n\omega t}{n\omega}\right]_0^1 - \frac{2}{n\omega}\int_0^1 t\sin n\omega t\,dt\right\} \\
&= 2\left\{-\frac{2}{n\omega}\left[t\frac{\cos n\omega t}{-n\omega}\right]_0^1 + \frac{2}{n\omega}\int_0^1 \cos n\omega t\,dt\right\} \\
&= -\frac{4}{n\omega}\left\{1\times\frac{\cos n\omega}{-n\omega} - \frac{1}{n\omega}\left[\frac{1}{n\omega}\sin n\omega t\right]_0^1\right\} \\
&= \frac{4}{n^2\omega^2}\cos n\omega = \frac{4}{n^2\pi^2}(-1)^n
\end{aligned}$$

$v(t)$は偶関数なので，$\boldsymbol{V}_{\mathrm{B}n} = 0$

したがって，

$$\begin{aligned}
v(t) &= \frac{1}{3} + \sum_{n=1}^{4}\frac{4}{n^2\pi^2}(-1)^n \cos n\omega t \\
&= \frac{1}{3} + \frac{4}{\pi^2}\left(-\cos\omega t + \frac{1}{4}\cos 2\omega t - \cdots\right)
\end{aligned}$$

（答）

3．

$$\boldsymbol{Z}_0 = j\omega L + \frac{R\dfrac{1}{j\omega C}}{R+\dfrac{1}{j\omega C}} = j\omega L + \frac{R}{1+j\omega CR}$$

$$= j\omega L + \frac{R(1-j\omega CR)}{1+(\omega CR)^2} = \frac{R}{1+(\omega CR)^2} + j\frac{\omega L\{1+(\omega CR)^2\}-\omega CR^2}{1+(\omega CR)^2}$$

$$\frac{\omega L\{1+(\omega CR)^2\}-\omega CR^2}{1+(\omega CR)^2} = 0 \text{ より}$$

$$L\{1+(\omega CR)^2\} = CR^2$$

$R \gg \dfrac{1}{\omega C}$ より，$\omega CR \gg 1$ なので

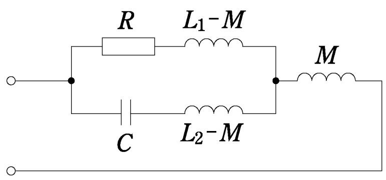
$$L(\omega CR)^2 = CR^2$$

$$\omega^2 CL = 1$$

$$\omega = \sqrt{\frac{1}{LC}}$$

$$f_0 = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \qquad \boldsymbol{Z}_0 = \frac{R}{1+(\omega CR)^2} = \frac{LR}{L+CR^2}$$ （答）

4．



$$\boldsymbol{Z} = j\omega M + \frac{\{R+j\omega(L_1-M)\}\left\{\dfrac{1}{j\omega C}+j\omega(L_2-M)\right\}}{R+j\omega(L_1-M)+\dfrac{1}{j\omega C}+j\omega(L_2-M)}$$ （答）