

## 5章 問題解答

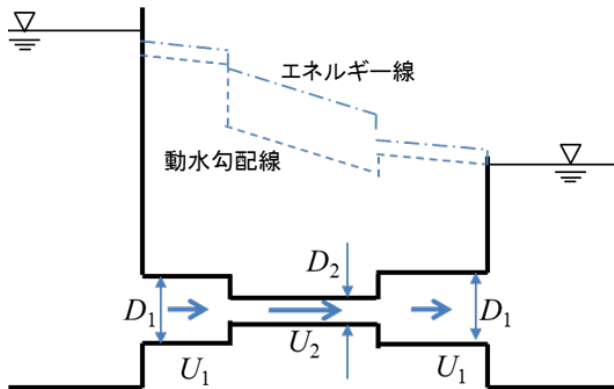
5-1

予習

- ①摩擦損失、②形状損失
- 省略(参考ページ:第4章管路の流れ(1), pp.112~113)
- 省略(参考ページ:第4章管路の流れ(1), pp.123~130)

演習問題 A

5-1-A1



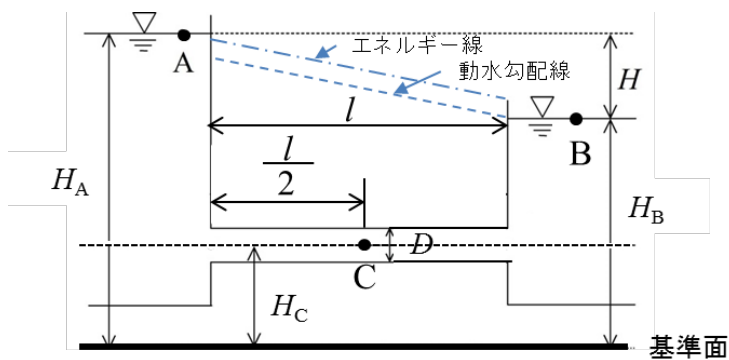
5-1-A2

$$(1) \quad H = \left( K_e + f \frac{l}{D} + K_o \right) \frac{U^2}{2g} \quad \text{より}$$

$$U = \sqrt{\frac{2gH}{K_e + f \frac{l}{D} + K_o}} = \sqrt{\frac{2 \times 9.8 \times (15 - 11)}{0.5 + 0.02 \times \frac{200}{0.3} + 1}} = 2.30 \text{ m/s}$$

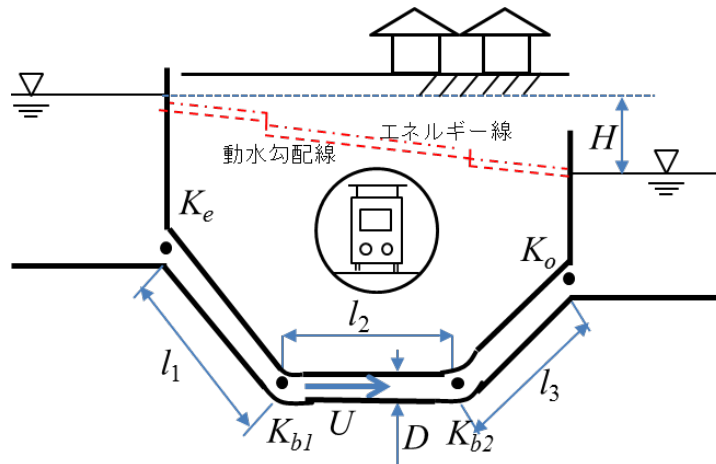
$$Q = \frac{\pi D^2}{4} U = \frac{\pi \times 0.3^2}{4} \times 2.30 = 0.163 \text{ m}^3/\text{s}$$

(2)





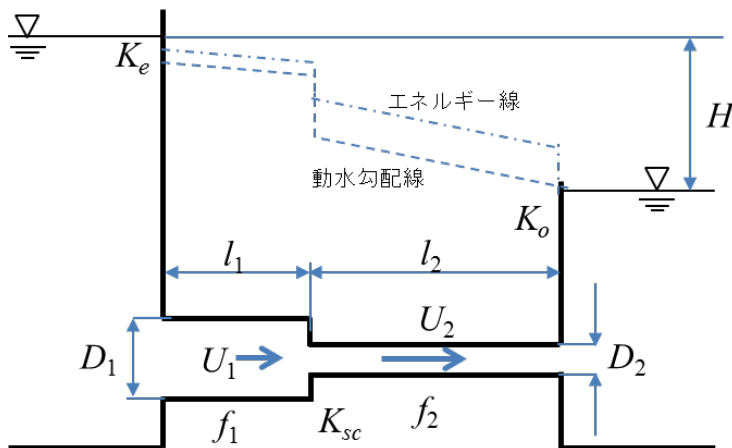
$$U = \sqrt{\frac{2gH}{(K_e + K_{b1} + K_{b2} + f \frac{l_1 + l_2 + l_3}{D} + K_o)}}$$



演習問題 B

5-1-B1

(1)



(2)

$$H = \left( K_e + f_1 \frac{l_1}{D_1} \right) \frac{U_1^2}{2g} + \left( K_{sc} + f_2 \frac{l_2}{D_2} + K_o \right) \frac{U_2^2}{2g}$$

(3)

$$U_1 \frac{\pi D_1^2}{4} = U_2 \frac{\pi D_2^2}{4} \text{ より、 } U_1 = \frac{D_2^2}{D_1^2} U_2$$

(4)

$$H = \left\{ \left( K_e + f_1 \frac{l_1}{D_1} \right) \frac{D_2^4}{D_1^4} + \left( K_{sc} + f_2 \frac{l_2}{D_2} + K_o \right) \right\} \frac{U_2^2}{2g} \text{ より、}$$

$$\begin{aligned}
 U_2 &= \sqrt{\frac{2gH}{\left(K_e + f_1 \frac{l_1}{D_1}\right) \frac{D_2^4}{D_1^4} + \left(K_{sc} + f_2 \frac{l_2}{D_2} + K_o\right)}} \\
 &= \sqrt{\frac{2 \times 9.8 \times 9}{\left(0.3 + 0.024 \frac{25}{1.2}\right) \frac{0.8^4}{1.2^4} + \left(0.25 + 0.024 \frac{40}{0.8} + 1\right)}} \\
 &= 8.22 \text{ m/s} \\
 Q &= \frac{\pi D_2^2}{4} U_2 = \frac{\pi \times 0.8^2}{4} \times 8.22 = 4.13 \text{ m}^3/\text{s}
 \end{aligned}$$

### 5-1-B2

(1)

$$H = f \frac{l_1 + l_2}{D} \frac{U^2}{2g} = 0.024 \times \frac{25 + 75}{0.6} \frac{U^2}{2g} = \frac{2U^2}{g}$$

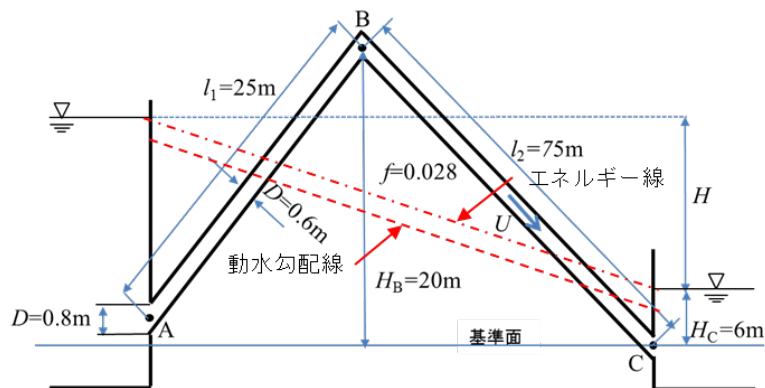
$$\frac{p_B}{\rho g} = H_C + H - \left(f \frac{l_1}{D} + 1\right) \frac{U^2}{2g} - H_B \geq -8$$

$$H - \left(f \frac{l_1}{D} + 1\right) \frac{U^2}{2g} \geq -8 - H_C + H_B = -8 - 6 + 20 = 6$$

$$H - \left(0.024 \frac{25}{0.6} + 1\right) \frac{U^2}{2g} = H - \frac{U^2}{g} = H - \frac{H}{2} = \frac{H}{2} \geq 6 \text{ より、}$$

$$H \geq 12 \text{ m}$$

(2)



(3)

$$U = \sqrt{\frac{gH}{2}} = \sqrt{\frac{9.8 \times 15}{2}} = 8.57 \text{ m/s}$$

(1)の第2式より,

$$\frac{p_B}{\rho g} = H_C + H - \left( f \frac{l_1}{D} + 1 \right) \frac{U^2}{2g} - H_B$$

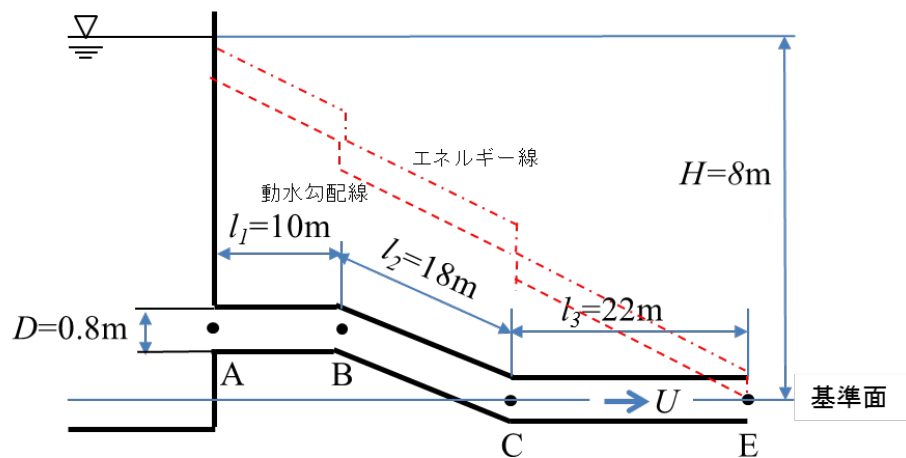
また,

$$\frac{U^2}{2g} = \frac{H}{4} \text{ より}$$

$$\begin{aligned} \frac{p_B}{\rho g} &= 6 + 15 - \left( 0.024 \times \frac{25}{0.6} + 1 \right) \times \frac{15}{4} - 20 \\ &= -6.5 \text{ m} \end{aligned}$$

### 5-1-B3

(1)



(2)

$$\begin{aligned} U &= \sqrt{\frac{2gH}{K_e + 2K_{be} + f \frac{l_1 + l_2 + l_3}{D} + 1}} \\ &= \sqrt{\frac{2 \times 9.8 \times 8}{0.56 + 2 \times 0.3 + 0.028 \frac{10 + 18 + 22}{0.8} + 1}} \\ &= 6.33 \text{ m/s} \end{aligned}$$

$$Q = \frac{\pi D^2}{4} U = \frac{\pi \times 0.8^2}{4} \times 6.33 = 3.18 \text{ m}^3/\text{s}$$

(3)

$$\begin{aligned} Q &= \frac{\pi D^2}{4} \sqrt{\frac{2gH}{K_e + 2K_{be} + f \frac{l_1 + l_2 + l_3}{D} + K_v + 1}} \\ &= \frac{\pi \times 0.8^2}{4} \sqrt{\frac{2 \times 9.8 \times 8}{0.56 + 2 \times 0.3 + 0.028 \frac{10 + 18 + 22}{0.8} + K_v + 1}} \\ &= \frac{6.294}{\sqrt{3.91 + K_v}} = 2.94 \end{aligned}$$

より、 $K_v = \left( \frac{6.294}{2.94} \right)^2 - 3.91 = 0.67$

(4)

$$U = \sqrt{\frac{2gH}{f \frac{l_1 + l_2 + l_3}{D} + 1}} = \sqrt{\frac{2 \times 9.8 \times 8}{0.028 \frac{10 + 18 + 22}{0.8} + 1}} = 7.55 \text{ m/s}$$

$$Q = \frac{\pi D^2}{4} U = \frac{\pi \times 0.8^2}{4} \times 7.55 = 3.80 \text{ m}^3/\text{s}$$

$$\begin{aligned} \frac{p_c}{\rho g} &= f \frac{l_3}{D} \frac{U^2}{2g} \\ &= 0.028 \times \frac{22}{0.8} \times \frac{7.55^2}{2 \times 9.8} \\ &= 2.24 \text{ m} \end{aligned}$$

## 5-2

### 予習

1. ① 摩擦 ② 形状 ③ 2 ④ 摩擦損失係数 ⑤ 長さ ⑥ 直径  
⑦ 急縮 ⑧ 入口 ⑨ 出口 ⑩ 曲がり ⑪  $K_i \frac{U^2}{2g}$

2. 省略(参考ページ:第4章管路の流れ(1), pp.118~119)

### 演習問題 A

#### 5-2-A1

$$h_L = f \frac{l}{D} \frac{U_1^2}{2g} = f \frac{2l}{D} \frac{U_2^2}{2g} \text{ より、 } U_2 = \frac{\sqrt{2}}{2} U_1$$

$$\frac{Q_2}{Q_1} = \frac{U_2}{U_1} = \frac{\sqrt{2}}{2}$$

### 5-2-A2

$$Q = Q_1 + Q_2 = \left(1 + \frac{\sqrt{2}}{2}\right) Q_1 = 0.3 \text{ より、 } Q_1 = \frac{0.3}{1 + \frac{\sqrt{2}}{2}} = 0.176 \text{ m}^3/\text{s}$$

$$Q_2 = Q - Q_1 = 0.3 - 0.176 = 0.124 \text{ m}^3/\text{s}$$

### 5-2-A3

$$H = f \frac{l/2}{D} \frac{U_1^2}{2g} + f \frac{l/2}{D} \frac{(U_1/2)^2}{2g} = f \frac{5}{16} \frac{l}{D} \frac{U_1^2}{g}$$

$$U_1 = \sqrt{\frac{16DgH}{5fl}} = \sqrt{\frac{16 \times 0.3 \times 9.8 \times 8.5}{5 \times 0.024 \times 800}} = 2.04 \text{ m/s}$$

$$Q = \frac{\pi D^2}{4} U_1 = \frac{\pi \times 0.3^2}{4} \times 2.04 = 0.144 \text{ m}^3/\text{s}$$

また、点 A より上流の流量は、

$$2U_1 \frac{\pi D^2}{4} = 2Q \text{ となるから、連続の式より穴から流出する流量は、}$$

$$2Q - Q = Q \text{ となる。}$$

## 演習問題 B

### 5-2-B1

(1)

$$h_L = f_1 \frac{l_1}{D_1} \frac{U_1^2}{2g} = f_2 \frac{l_2}{D_2} \frac{U_2^2}{2g} \text{ より、 } U_2 = \sqrt{\frac{f_1 l_1 D_2}{f_2 l_2 D_1}} U_1$$

$$Q = \frac{\pi D_1^2}{4} U_1 + \frac{\pi D_2^2}{4} U_2 = \frac{\pi D_1^2}{4} U_1 + \frac{\pi D_1^2}{4} \frac{D_2^2}{D_1^2} \sqrt{\frac{f_1 l_1 D_2}{f_2 l_2 D_1}} U_1$$

$$= \left(1 + \frac{D_2^2}{D_1^2} \sqrt{\frac{f_1 l_1 D_2}{f_2 l_2 D_1}}\right) \frac{\pi D_1^2}{4} U_1 = \left(1 + \sqrt{\frac{f_1 l_1 D_2^5}{f_2 l_2 D_1^5}}\right) Q_1$$

$$Q_1 = \frac{Q}{\left(1 + \sqrt{\frac{f_1 l_1 D_2^5}{f_2 l_2 D_1^5}}\right)}$$

同様に、 $Q_2 = \frac{Q}{\left(1 + \sqrt{\frac{f_2 l_2 D_1^5}{f_1 l_1 D_2^5}}\right)}$

(2)

$$f_1 = \frac{124.5n^2}{D_1^{\frac{1}{3}}} = \frac{124.5 \times 0.012^2}{0.2^{\frac{1}{3}}} = 0.0307$$

$$f_2 = \frac{124.5n^2}{D_2^{\frac{1}{3}}} = \frac{124.5 \times 0.012^2}{0.1^{\frac{1}{3}}} = 0.0386$$

$$Q_1 = \frac{Q}{\left(1 + \sqrt{\frac{f_1 l_1 D_2^5}{f_2 l_2 D_1^5}}\right)} = \frac{0.3}{\left(1 + \sqrt{\frac{0.0307 \times 120 \times 0.1^5}{0.0386 \times 30 \times 0.2^5}}\right)} = 0.228 \text{ m}^3/\text{s}$$

$$Q_2 = \frac{Q}{\left(1 + \sqrt{\frac{f_2 l_2 D_1^5}{f_1 l_1 D_2^5}}\right)} = \frac{0.3}{\left(1 + \sqrt{\frac{0.0386 \times 30 \times 0.2^5}{0.0307 \times 120 \times 0.1^5}}\right)} = 0.072 \text{ m}^3/\text{s}$$

(3)

$$\frac{Q_1}{Q_2} = \frac{\left(1 + \sqrt{\frac{f_1 l_2 D_1^5}{f_1 l_1 D_1^5}}\right)}{\left(1 + \sqrt{\frac{f_1 l_1 D_1^5}{f_1 l_2 D_1^5}}\right)} = \frac{\left(1 + \sqrt{\frac{l_2}{l_1}}\right)}{\left(1 + \sqrt{\frac{l_1}{l_2}}\right)} = \frac{\left(1 + \sqrt{\frac{30}{120}}\right)}{\left(1 + \sqrt{\frac{120}{30}}\right)} = 0.5$$

## 5-2-B2

$$(1) H = f \frac{l U_1^2}{D 2g} = \frac{fl}{2gD} U_1^2 = \frac{fl}{2gD} \left(\frac{Q_1}{\frac{\pi D^2}{4}}\right)^2 = \frac{8fl}{\pi^2 g D^5} Q_1^2$$

$$(2) H_B - H = f \frac{l U_2^2}{D 2g} = \frac{fl}{2gD} U_2^2 = \frac{fl}{2gD} \left(\frac{Q_2}{\frac{\pi D^2}{4}}\right)^2 = \frac{8fl}{\pi^2 g D^5} Q_2^2$$

$$(3) H_C - H = f \frac{l U_3^2}{D 2g} = \frac{fl}{2gD} U_3^2 = \frac{fl}{2gD} \left(\frac{Q_3}{\frac{\pi D^2}{4}}\right)^2 = \frac{8fl}{\pi^2 g D^5} Q_3^2$$

$$(4) Q_1 = Q_2 + Q_3$$

$$(5) H = \frac{8fl}{\pi^2 gD^5} (Q_2 + Q_3)^2$$

$$H_B = \frac{8fl}{\pi^2 gD^5} Q_2^2 + H = \frac{8fl}{\pi^2 gD^5} (2Q_2^2 + 2Q_2Q_3 + Q_3^2)$$

$$H_C = \frac{8fl}{\pi^2 gD^5} Q_3^2 + H = \frac{8fl}{\pi^2 gD^5} (Q_2^2 + 2Q_2Q_3 + 2Q_3^2) \quad \text{より、}$$

$$K_1 = \frac{8fl}{\pi^2 gD^5} \quad \text{とおくと、}$$

$$H_C - H_B = K_1 (Q_3^2 - Q_2^2)$$

$$\text{ゆえに、} Q_3 = \sqrt{\frac{H_C - H_B}{K_1} + Q_2^2}$$

$$(6) \frac{H_C - H_B}{K_1} = K_2 \quad \text{において、} H_B \text{の式を書き換えると}$$

$$H_B = K_1 (2Q_2^2 + 2Q_2Q_3 + Q_3^2) = K_1 (2Q_2^2 + 2Q_2\sqrt{K_2 + Q_2^2} + K_2 + Q_2^2)$$

$$\text{ここで、} \text{func}(Q_2) = K_1 (3Q_2^2 + 2Q_2\sqrt{K_2 + Q_2^2} + K_2) - H_B = 0$$

$$\text{func}'(Q_2) = 6K_1Q_2 + 2K_1\sqrt{K_2 + Q_2^2} + \frac{2K_1Q_2^2}{\sqrt{K_2 + Q_2^2}} \quad \text{とおく。}$$

これをニュートン・ラフソン法で解く。ただし、

$$K_1 = \frac{8fl}{\pi^2 gD^5} = \frac{8 \times 0.026 \times 120}{\pi^2 \times 9.8 \times 0.25^5} = 264.3$$

$$K_2 = \frac{H_C - H_B}{K_1} = \frac{18 - 12}{264.3} = 0.0227$$

計算手順は以下のとおり。

$Q_2$	$\text{func}(Q_2)$	$\text{func}''(Q_2)$	$Q_2 - \frac{\text{func}(Q_2)}{\text{func}'(Q_2)}$
0.200	52.179	533.868	0.102
0.102	12.133	288.732	0.060
0.060	2.044	193.099	0.050
0.050	0.119	170.798	0.049
0.049	0.000	169.363	0.049

$$Q_2 = 0.049 \text{ m}^3/\text{s}$$

$$Q_3 = \sqrt{K_2 + Q_2^2} = \sqrt{0.0227 + 0.049^2} = 0.158 \text{ m}^3/\text{s}$$

$$Q_1 = Q_2 + Q_3 = 0.049 + 0.158 = 0.207 \text{ m}^3/\text{s}$$

### 5-2-B3

$$H_1 = \frac{8fl_1}{\pi^2 gD^5} Q_1^2 = \frac{8 \times 0.022 \times 300}{\pi^2 \times 9.8 \times 0.5^5} Q_1^2 = 17.47 Q_1^2 = K_1 Q_1^2$$

$$H_2 = \frac{8fl_2}{\pi^2 gD^5} Q_2^2 = \frac{8 \times 0.022 \times 280}{\pi^2 \times 9.8 \times 0.5^5} Q_2^2 = 16.30 Q_2^2 = K_2 Q_2^2$$

$$H_3 = \frac{8fl_3}{\pi^2 gD^5} Q_3^2 = \frac{8 \times 0.022 \times 450}{\pi^2 \times 9.8 \times 0.5^5} Q_3^2 = 26.20 Q_3^2 = K_3 Q_3^2$$

ハーディークロス法を用いると、

修正回数	管路 <i>i</i>	仮定流量 <i>Q<sub>i</sub></i>	<i>K<sub>i</sub>Q<sub>i</sub><sup>2</sup></i>	<i>K<sub>i</sub>Q<sub>i</sub></i>	$Q - \frac{\sum K_i Q_i^2}{\sum 2K_i Q}$
I	1	2.2	84.5548	38.434	2.255
	2	0.8	-10.432	13.04	0.745
	3	1.8	-84.888	47.16	1.745
	Σ		-10.7652	98.634	
II	1	2.255	88.802	39.387	2.255
	2	0.745	-9.057	12.150	0.745
	3	1.745	-79.819	45.730	1.745
	Σ		-0.07	97.27	

したがって、

$$Q_1 = 2.255 \text{ m}^3/\text{s} \quad (\text{A} \rightarrow \text{B})$$

$$Q_2 = 0.745 \text{ m}^3/\text{s} \quad (\text{A} \rightarrow \text{C})$$

$$Q_3 = 1.745 \text{ m}^3/\text{s} \quad (\text{C} \rightarrow \text{B})$$

### 5-3

#### 予習

1. 省略

2. ① ポンプ                      ② 水力発電

3.

ポンプ動力は単位時間内に汲み上げる水の重量を水面から高所へ移動させるための仕事であるから、まず水を満たした水槽内の水の質量 *m* を求めると、

$$m = 1 \times 0.7 \times 0.5 \times 1000 \text{ kg} = 350 \text{ kg}$$

となる。また、水槽の底から水を注入するとすれば、移動の高さは水面から水槽の重心までの距離となるから、

$$h=30+0.5/2=30.25\text{m}$$

したがって、仕事は  $mgh$  で与えられるので、 $T=6$  分間でこれだけの仕事を行ったとすると、動力は以下ようになる。

$$P=mgh/T=350\times 9.8\times 30.25/(6\times 60)=288\text{W}=0.288\text{kW}$$

### 演習問題 A

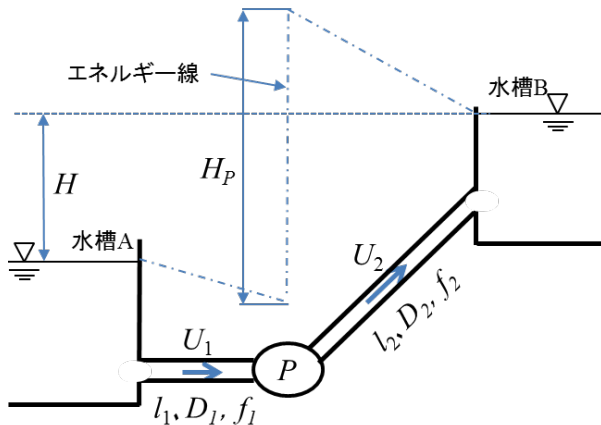
#### 5-3-A1

- ① ポテンシャル ② 運動 ③ 水車 ④ 発電機 ⑤ 水力発電  
⑥ 機械的 ⑦ 揚水 ⑧ ポンプ

### 演習問題 B

#### 5-3-B1

(1)



(2)

$$\begin{aligned} H_p &= H + f_1 \frac{l_1}{D_1} \frac{U_1^2}{2g} + f_2 \frac{l_2}{D_2} \frac{U_2^2}{2g} \\ &= H + f_1 \frac{l_1}{D_1} \frac{(4Q/\pi D_1^2)^2}{2g} + f_2 \frac{l_2}{D_2} \frac{(4Q/\pi D_2^2)^2}{2g} \\ &= H + \left( \frac{8f_1 l_1}{\pi^2 g D_1^5} + \frac{8f_2 l_2}{\pi^2 g D_2^5} \right) Q^2 \end{aligned}$$

(3)

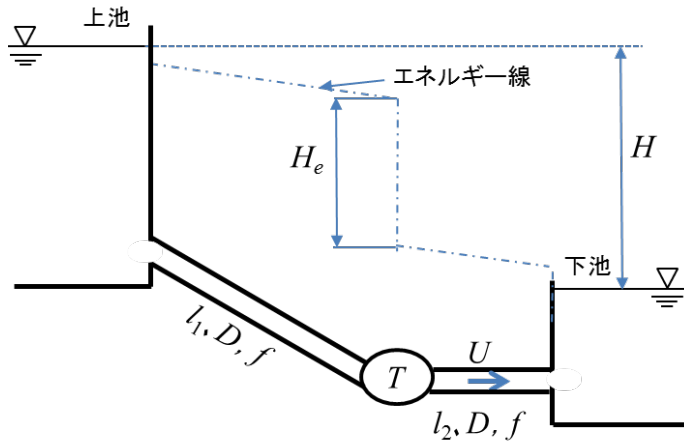
$$\begin{aligned} H_p &= H + \left( \frac{8f_1 l_1}{\pi^2 g D_1^5} + \frac{8f_2 l_2}{\pi^2 g D_2^5} \right) Q^2 \\ &= 7 + \left( \frac{8 \times 0.022 \times 10}{\pi^2 \times 9.8 \times 0.5^5} + \frac{8 \times 0.024 \times 14}{\pi^2 \times 9.8 \times 0.4^5} \right) \times 0.4^2 \\ &= 7.527\text{m} \end{aligned}$$

$$P_p = \frac{\rho g Q H_p}{\eta_p} = \frac{1000 \times 9.8 \times 0.4 \times 7.527}{0.74}$$

$$= 39900 \text{ W} = 39.9 \text{ kW}$$

### 5-3-B2

(1)



$$(2) \quad H = H_T + \left( f_e + f \frac{l_1 + l_2}{D} + 1 \right) \frac{U^2}{2g} = H_T + \frac{8Q^2}{\pi^2 g D^4} \left( f_e + f \frac{l_1 + l_2}{D} + 1 \right)$$

$$= H_T + \frac{8 \times 2.8^2}{\pi^2 \times 9.8 \times 1.2^4} \left( 0.6 + 0.022 \times \frac{500 + 200}{1.2} + 1 \right)$$

$$= H_T + 0.3127 \times 14.43 = 45$$

$$H_T = 45 - 4.512 = 40.49 \text{ m}$$

$$P_T = \eta_T \rho g Q H_p = 0.8 \times 1000 \times 9.8 \times 2.8 \times 40.49$$

$$= 889000 \text{ W} = 889 \text{ kW}$$