表 6-1 簡単な図形の重心の導出

線分

$$x_G = \frac{\int_0^l x dx}{l} = \frac{\left[\frac{1}{2}x^2\right]_0^l}{l} = \frac{\frac{1}{2}l^2}{l} = \frac{l}{2}$$

円弧

$$y_G = \frac{\int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} r\cos\theta \, r d\theta}{r\alpha} = \frac{r^2[\sin\theta]_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}}}{r\alpha} = \frac{2r^2\sin\frac{\alpha}{2}}{r\alpha} = \frac{2r}{\alpha}\sin\frac{\alpha}{2}$$

三角形

底辺の長さをbとすると，

$$y_G = \frac{\int_0^h y\left(b - \frac{b}{h}y\right)dy}{\int_0^h \left(b - \frac{b}{h}y\right)dy} = \frac{\frac{bh^2}{2} - \frac{bh^2}{3}}{bh - \frac{bh}{2}} = \frac{\frac{bh^2}{6}}{\frac{bh}{2}} = \frac{1}{3}h$$

平行四辺形

底辺の長さをbとすると，

$$y_G = \frac{\int_0^h y \, bdy}{\int_0^h bdy} = \frac{\frac{bh^2}{2}}{bh} = \frac{1}{2}h$$

台形

$$y_G = \frac{\int_0^h y\left(\frac{ah + (b-a)y}{h}\right)dy}{\int_0^h \left(\frac{ah + (b-a)y}{h}\right)dy} = \frac{\frac{(a+2b)h^2}{6}}{\frac{(a+b)h}{2}} = \frac{1}{3}\left(\frac{a+2b}{a+b}\right)h$$

扇形

$$y_G = \frac{\int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} \left\{\int_0^r r\cos\theta \, dr\right\} rd\theta}{\int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} \left\{\int_0^r dr\right\} rd\theta} == \frac{\frac{2r^3}{3}\sin\frac{\alpha}{2}}{\frac{r^2}{2}\alpha} = \frac{4r}{3\alpha}\sin\frac{\alpha}{2}$$

半円

$$y_G = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\{ \int_0^r r\cos\theta \, dr \right\} r d\theta}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left\{ \int_0^r dr \right\} r d\theta} == \frac{\frac{2r^3}{3}\sin\frac{\pi}{2}}{\frac{r^2}{2}\pi} = \frac{4r}{3\pi}$$

円すい

$$y_G = \frac{\int_V y \, dV}{\int_V dV} = \frac{\int_0^h y \, \pi \frac{(h-y)^2 r^2}{h^2} dy}{\int_0^h \pi \frac{(h-y)^2 r^2}{h^2} dy} = \frac{\frac{\pi r^2 h^2}{12}}{\frac{\pi r^2 h}{3}} = \frac{1}{4} h$$

角すい

$$y_G = \frac{\int_V y \, dV}{\int_V dV} = \frac{\int_0^h y \, A \frac{(h-y)^2}{h^2} dy}{\int_0^h A \frac{(h-y)^2}{h^2} dy} = \frac{1}{4} h$$

半球

$$y_G = \frac{\int_V y \, dV}{\int_V dV} = \frac{\int_0^r y \, \pi \left(r^2 - y^2\right) dy}{\int_0^h \pi \left(r^2 - y^2\right) dy} = \frac{\frac{\pi r^4}{4}}{\frac{2\pi r^3}{3}} = \frac{3}{8} r$$