２章　２次関数とグラフ，方程式・不等式　解答

１節　２次方程式

練習１

⑴ $x^2 = 4$

$x = \pm 2$

⑵ $2x^2 - 10 = 0$

$2x^2 = 10$

$x^2 = 5$

$x = \pm\sqrt{5}$

⑶ $(x-2)^2 = 3$

$(x-2) = \pm\sqrt{3}$

$x = 2 \pm \sqrt{3}$

練習２

⑴ $x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2,\ 3$

⑵ $2x^2 - 4x = 0$

$2x(x-2) = 0$

$x = 0,\ 2$

⑶ $2x^2 + 5x + 3 = 0$

$(x+1)(2x+3) = 0$

$x = -1,\ -\dfrac{3}{2}$

⑷ $6x^2 + 7x - 3 = 0$

$(2x+3)(3x-1) = 0$

$x = -\dfrac{3}{2},\ \dfrac{1}{3}$

練習３

⑴ $x^2 + x - 3 = 0$

$$x = \frac{-1 \pm \sqrt{1+12}}{2} = \frac{-1 \pm \sqrt{13}}{2}$$

⑵ $3x^2 - 5x - 1 = 0$

$$x = \frac{5 \pm \sqrt{25+12}}{6} = \frac{5 \pm \sqrt{37}}{6}$$

⑶ $2x^2 - 10x + \dfrac{25}{2} = 0$

$4x^2 - 20x + 25 = 0$

$$x = \frac{20 \pm \sqrt{400 - 400}}{8} = \frac{5}{2}$$

⑷ $-4x^2 - 5x + 6 = 0$

$$x = \frac{5 \pm \sqrt{25+96}}{-8} = \frac{5 \pm 11}{-8} = -2,\quad \frac{3}{4}$$

⑸ $x^2 - x + 1 = 0$

$x^2 - x + 1 = 0$

$$x = \frac{1 \pm \sqrt{1-4}}{2}$$

$$= \frac{1 \pm \sqrt{3}i}{2}$$

⑹ $8x^2 + 4x + 3 = 0$

$$x = \frac{-4 \pm \sqrt{16-96}}{16}$$

$$= \frac{-4 \pm \sqrt{80}\,i}{16}$$

$$= \frac{-4 \pm 4\sqrt{5}\,i}{16}$$

$$= \frac{-1 \pm \sqrt{5}\,i}{4}$$

練習 4

⑴ $x^2+5x+5=0$

$D=25-4.5=20>0$

よって，異なる2つの実数解をもつ

⑵ $2x^2-4x+3=0$

$D=16-4-2-3=-8<0$

よって，2つの共役な虚数解をもつ

⑶ $-x^2+2\sqrt{3}\,x-3=0$

$D=12-12=0$

よって，重解をもつ

練習 5

⑴ $2x^2+2kx-k+4=0$

$D=(2k)^2-4\cdot2\cdot(-k+4)=0$

$4k^2+8k-3^2=0$

$k^2+2k-8=0$

$(k+4)(k-2)=0$

$\therefore\ k=2,\ -4$

$k=2$ のとき，$2x^2+4x+2=0$

$2(x+1)^2=0$

$\therefore$ 重解は $x=-1$

$k=-4$ のとき，$2x^2-8x-8=0$

$2(x-2)^2=0$

$\therefore$ 重解は $x=2$

練習 6

⑴ $x^2+2x+5=0$

$\alpha+\beta=-2,\ \alpha\beta=5$

和 $=-2$，　積 $=5$

⑵ $3x^2-7x+2=0$

$\alpha+\beta=\dfrac{7}{3}$，$\alpha\beta=\dfrac{2}{3}$

和 $=\dfrac{7}{3}$，　積 $=\dfrac{2}{3}$

⑶ $6x^2+3x-4=0$

$\alpha+\beta=-\dfrac{1}{2}$，$\alpha\beta=-\dfrac{2}{3}$

和 $=-\dfrac{1}{2}$，　積 $=-\dfrac{2}{3}$

⑷ $2x^2+5=0$

$\alpha+\beta=0,\ \alpha\beta=\dfrac{5}{2}$

和 $=0$，　積 $=\dfrac{5}{2}$

⑸ 和 $=\dfrac{2}{5}$，　積 $=0$

⑹ 和 $=-\dfrac{3}{2}$，　積 $=-\dfrac{1}{4}$

練習７

$$3x^2-2x+1=0 \qquad \alpha+\beta=\frac{2}{3},\ \alpha\beta=\frac{1}{3}$$

⑴ $(\alpha+1)(\beta+1)=\alpha\beta+(\alpha+\beta)+1=\frac{1}{3}+\frac{2}{3}+1=2$

⑵ $\frac{\beta}{\alpha}+\frac{\alpha}{\beta}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{\left(\frac{2}{3}\right)^2-2\cdot\frac{1}{3}}{\frac{1}{3}}=\frac{\frac{4}{9}-\frac{2}{3}}{3}=\frac{4}{3}-2=-\frac{2}{3}$

⑶ $\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)=\frac{2}{3}\left(\left(\frac{2}{3}\right)^2-3\frac{1}{3}\right)=\frac{2}{3}\left(\frac{4}{9}-1\right)=\frac{2}{3}\left(\frac{-5}{9}\right)=-\frac{10}{27}$

練習８

２つの解は２αと３αと表せる。

解と係数の関係から

$2\alpha+3\alpha=15$……①

$2\alpha\cdot 3\alpha=k$……②

①より　$\alpha=3$　このとき，②より $k=54$

よって，$k=54,\ x=6,\ 9$

練習９

⑴ $x^2+4x-2=0$ を解くと

$$x=\frac{-4\pm\sqrt{16+8}}{2}=\frac{-4\pm 2\sqrt{6}}{2}=-2\pm\sqrt{6}$$

$\therefore\ x^2+4x-2=\left\{x-\left(-2+\sqrt{6}\right)\right\}\left\{x-\left(-2-\sqrt{6}\right)\right\}=\left(x+2-\sqrt{6}\right)\left(x+2+\sqrt{6}\right)$

⑵ $x^2-x+1=0$ を解くと

$x=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm\sqrt{3}\,i}{2}$　　$\therefore\ x^2-x+1=\left(x-\frac{1+\sqrt{3}\,i}{2}\right)\left(x-\frac{1-\sqrt{3}\,i}{2}\right)$

⑶ $4x^2+1=0$ を解くと

$x=\pm\frac{i}{2}$　　$\therefore\ 4x^2+1=4\left(x-\frac{i}{2}\right)\left(x+\frac{i}{2}\right)$

⑷ $5x^2+2x+1=0$ を解くと

$x=\frac{-2\pm\sqrt{4-20}}{10}=\frac{-2\pm 4i}{10}=\frac{-1\pm 2i}{5}$　　$\therefore\ 5x^2+2x+1=5\left(x-\frac{-1+2i}{5}\right)\left(x-\frac{-1-2i}{5}\right)$

節末問題

1．

⑴ $x^2-8x+7=0$

$(x-7)(x-1)=0$

$x=1,\ 7$

⑵ $2x^2-4x=x^2+12$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$x=-2,\ \ 6$

⑶ $0.2x^2+0.5x-1.2=0$

$2x^2+5x-12=0$

$(x+4)(2x-3)=0$

$x=-4,\ \dfrac{3}{2}$

⑷ $x^2+\dfrac{x}{12}-\dfrac{5}{8}=0$

$24x^2+2x-15=0$

$(6x+5)(4x-3)=0$

$x=-\dfrac{5}{6},\ \dfrac{3}{4}$

⑸ $x^2+3\sqrt{2}x+4=0$

$$x=\frac{-3\sqrt{2}\pm\sqrt{18-16}}{2}$$

$$=\frac{-3\sqrt{2}\pm\sqrt{2}}{2}=-\frac{2\sqrt{2}}{2},\ -\frac{4\sqrt{2}}{2}$$

$$=-\sqrt{2},\ -2\sqrt{2}$$

⑹ $(x+3)^2-2(x+3)-2=0$

$x+3=t$とおくと

$t^2-2t-2=0,\ t=\dfrac{2\pm\sqrt{4+8}}{2}=1\pm\sqrt{3}$

$x+3=1\pm\sqrt{3}$

$\therefore\ x=-2\pm\sqrt{3}$

2．

$x^2+2kx-k+2=0$

$D=(2k)^2-4\cdot(-k+2)=0$

$4k^2+4k-8=0$

$k^2+k-2=0$

$(k+2)(k-1)=0$

$k=1,\ -2$

$k=1$ のとき，$x^2+2x+1=0$

$(x+1)^2=0$

$x=-1$ で負の重解だから不適

$k=-2$ のとき，$x^2-4x+4=0$

$(x-2)^2=0$

重解は $x=2$

3．$x^2-4x-2=0$

$\alpha+\beta=4,\ \alpha\beta=-2$

⑴ $(\alpha+2\beta)(\beta+2\alpha)=\alpha\beta+2\alpha^2+2\beta^2+4\alpha\beta=\alpha\beta+2(\alpha+\beta)^2=-2+2\cdot4^2=-2+32=30$

⑵ $(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2=(\alpha+\beta)^2-4\alpha\beta=4^2-4\cdot(-2)=16+8=24$

⑶ $$\frac{\beta^2}{\alpha}+\frac{\alpha^2}{\beta}=\frac{\beta^3+\alpha^3}{\alpha\beta}=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$=\frac{4^3-3\cdot(-2)\cdot4}{\alpha\beta}=\frac{88}{-2}=-44$$

4． $6x^2 - 2x + 3 = 0 \quad \alpha + \beta = \dfrac{1}{3} \quad \alpha\beta = \dfrac{1}{2}$

⑴ $(x - (\alpha - 1))(x - (\beta - 1)) = 0$

$x^2 - (\alpha + \beta - 2)x + (\alpha\beta - \alpha - \beta + 1) = 0$

$x^2 - \left(\dfrac{1}{3} - 2\right)x + \left(\dfrac{1}{2} - \dfrac{1}{3} + 1\right) = 0$

$x^2 + \dfrac{5}{3}x + \dfrac{7}{6} = 0 \qquad 6x^2 + 10x + 7 = 0$

⑵ $\left(x - \dfrac{2}{\alpha}\right)\left(x - \dfrac{2}{\beta}\right) = 0$

$x^2 - \left(\dfrac{2}{\alpha} + \dfrac{2}{\beta}\right)x + \dfrac{4}{\alpha\beta} = 0$

$x^2 - \left(\dfrac{2(\alpha + \beta)}{\alpha\beta}\right)x + \dfrac{4}{\alpha\beta} = 0$

$x^2 - \dfrac{2 \cdot \frac{1}{3}}{\frac{1}{2}}x + \dfrac{4}{\frac{1}{2}} = 0$

$x^2 - \dfrac{4}{3}x + 8 = 0$

$3x^2 - 4x + 24 = 0$

5．

⑴ $x^2 - 2x + 4 = 0$

$x = \dfrac{2 \pm \sqrt{4 - 16}}{2} = \dfrac{2 \pm 2\sqrt{3}\,i}{2} = 1 \pm \sqrt{3}\,i$

⑵

$$
\begin{array}{r l}
 & x - 2 \\
x^2 - 2x + 4 \;\big) & x^3 - 4x^2 + 6x - 3 \\
 & \underline{x^3 - 2x^2 + 4x \quad\;\;} \\
 & \quad -2x^2 + 2x - 3 \\
 & \quad \underline{-2x^2 + 4x - 8} \\
 & \qquad\quad -2x + 5
\end{array}
$$

$\therefore\ x^2 - 4x^2 + 6x = 3 = (x^2 - 2x + 4)(x - 2) - 2x + 5$

⑶ $\alpha = 1 \pm \sqrt{3}\,i$

（ｉ） $\alpha^2 - 2\alpha + 4 = 0$

（ⅱ） $\alpha^3 - 4\alpha^2 + 6\alpha - 3 = (\alpha^2 - 2\alpha + 4)(\alpha - 2) - 2\alpha + 5$

$\alpha = 1 + \sqrt{3}\,i$ を代入すると

与式 $= 0 \cdot (-1 + \sqrt{3}\,i - 2) - 2(1 + \sqrt{3}\,i) + 5$

$= 3 - 2\sqrt{3}\,i$

6． $x^2 - 4x + k - 2 = 0$ の異なる2つの解をα，βとすると

$\alpha + \beta = 4 \quad \alpha\beta = k - 2$

$D = (-4)^2 - 4(k - 2) > 0$ ……①

$\alpha + \beta = 4 > 0$ ……②

$\alpha\beta = k - 2 > 0$ ……③

①～③を同時に満たす k の値の範囲を求めればよい。

①より $k < 6$，③より $k > 2$ $\quad \therefore 2 < k < 6$