新版微分積分演習 解答

1章 微分法 I

1節　関数の極限

1

(1) $\lim_{x\to 2}(2x^2-3x-4)=2\cdot 2^2-3\cdot 2-4=-2$

(2) $\lim_{x\to -1}(x^3-2x+2)=(-1)^3-2\cdot(-1)+2=3$

(3) $\lim_{x\to 1}\sqrt{3x-1}=\sqrt{3\cdot 1-1}=\sqrt{2}$

(4) $\lim_{x\to -2}\sqrt{2-x}=\sqrt{2-(-2)}=2$

2

(1) $\lim_{x\to 0}\dfrac{x^2+2x}{2x}=\lim_{x\to 0}\dfrac{\cancel{x}(x+2)}{2\cancel{x}}=\lim_{x\to 0}\dfrac{x+2}{2}=1$

(2) $\lim_{x\to 3}\dfrac{x^2-9}{x-3}=\lim_{x\to 3}\dfrac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}=\lim_{x\to 3}(x+3)=6$

(3) $\lim_{x\to -2}\dfrac{x^2-x-6}{x+2}=\lim_{x\to -2}\dfrac{\cancel{(x+2)}(x-3)}{\cancel{x+2}}=\lim_{x\to -2}(x-3)=-5$

3

(1) $\lim_{x\to 1}\dfrac{x^2+x-2}{x^2-x}=\lim_{x\to 1}\dfrac{(x+2)\cancel{(x-1)}}{x\cancel{(x-1)}}=\lim_{x\to 1}\dfrac{x+2}{x}=3$

(2) $\lim_{x\to -1}\dfrac{x^3+1}{x+1}=\lim_{x\to -1}\dfrac{\cancel{(x+1)}(x^2-x+1)}{\cancel{x+1}}=\lim_{x\to -1}(x^2-x+1)=3$

(3) $\lim_{x\to 2}\dfrac{x^3-3x^2+4}{x^2-4x+4}=\lim_{x\to 2}\dfrac{(x+1)\cancel{(x-2)^2}}{\cancel{(x-2)^2}}=\lim_{x\to 2}(x+1)=3$

4

(1) $\lim_{x\to 0}\dfrac{1}{x}\left(1-\dfrac{1}{x+1}\right)=\lim_{x\to 0}\left(\dfrac{1}{\cancel{x}}\cdot\dfrac{\cancel{x}}{x+1}\right)=\lim_{x\to 0}\dfrac{1}{x+1}=1$

(2) $\lim_{x\to 0}\dfrac{1}{x}\left(2+\dfrac{4}{x-2}\right)=\lim_{x\to 0}\left(\dfrac{1}{\cancel{x}}\cdot\dfrac{2\cancel{x}}{x-2}\right)=-1$

5

(1) $\lim_{x\to 2}\dfrac{\sqrt{x-1}-1}{x-2}=\lim_{x\to 2}\dfrac{(\sqrt{x-1}-1)(\sqrt{x-1}+1)}{(x-2)(\sqrt{x-1}+1)}=\lim_{x\to 2}\dfrac{\cancel{x-2}}{\cancel{(x-2)}(\sqrt{x-1}+1)}=\lim_{x\to 2}\dfrac{1}{\sqrt{x-1}+1}$

$=\dfrac{1}{2}$

(2) $\lim_{x\to -2}\dfrac{\sqrt{x+11}-3}{x+2}=\lim_{x\to -2}\dfrac{(\sqrt{x+11}-3)(\sqrt{x+11}+3)}{(x+2)(\sqrt{x+11}+3)}=\lim_{x\to -2}\dfrac{\cancel{x+2}}{\cancel{(x+2)}(\sqrt{x+11}+3)}$

$=\lim_{x\to -2}\dfrac{1}{\sqrt{x+11}+3}=\dfrac{1}{6}$

(3) $\lim_{x\to 1}\dfrac{2\sqrt{x}-\sqrt{3x+1}}{x-1}=\lim_{x\to 1}\dfrac{(2\sqrt{x}-\sqrt{3x+1})(2\sqrt{x}+\sqrt{3x+1})}{(x-1)(2\sqrt{x}+\sqrt{3x+1})}=\lim_{x\to 1}\dfrac{\cancel{x-1}}{\cancel{(x-1)}(2\sqrt{x}+\sqrt{3x+1})}$

$=\lim_{x\to 1}\dfrac{1}{2\sqrt{x}+\sqrt{3x+1}}=\dfrac{1}{4}$

(4) $\lim_{x\to 3}\dfrac{\sqrt{3x}-\sqrt{2x+3}}{x-3}=\lim_{x\to 3}\dfrac{(\sqrt{3x}-\sqrt{2x+3})(\sqrt{3x}+\sqrt{2x+3})}{(x-3)(\sqrt{3x}+\sqrt{2x+3})}=\lim_{x\to 3}\dfrac{\cancel{x-3}}{\cancel{(x-3)}(\sqrt{3x}+\sqrt{2x+3})}$

$=\lim_{x\to 3}\dfrac{1}{\sqrt{3x}+\sqrt{2x+3}}=\dfrac{1}{6}$

6

(1) 与式が成り立つには　$\lim_{x\to 1}(x^2+ax+b)=0$　$\therefore$　$1+a+b=0$　$\therefore$　$b=-a-1$　$\cdots$ ①

$\therefore$　$\lim_{x\to 1}\dfrac{x^2+ax+b}{x-1}=\lim_{x\to 1}\dfrac{x^2+ax+(-a-1)}{x-1}=\lim_{x\to 1}\dfrac{\cancel{(x-1)}(x+a+1)}{\cancel{x-1}}=\lim_{x\to 1}(x+a+1)=4$

$\therefore$　$a+2=4$　$\therefore$　$a=2$　これを ① に代入して　$b=-3$

以上より　$a=2,\ b=-3$

(2) 与式が成り立つには $\lim_{x \to 2}(x^2+ax+b)=0$ $\quad\therefore\quad 4+2a+b=0$ $\quad\therefore\quad b=-2a-4 \quad \cdots$ ①

$\therefore\quad \lim_{x\to 2}\dfrac{x^2+ax+b}{x-2}=\lim_{x\to 2}\dfrac{x^2+ax+(-2a-4)}{x-2}=\lim_{x\to 2}\dfrac{\cancel{(x-2)}(x+a+2)}{\cancel{x-2}}=\lim_{x\to 2}(x+a+2)=-1$

$\therefore\quad a+4=-1 \quad\therefore\quad a=-5$ 　これを ① に代入して $b=6$

以上より $a=-5,\ b=6$

(3) 与式が成り立つには $\lim_{x\to -2}\left(a\sqrt{x+3}+b\right)=0$ $\quad\therefore\quad a+b=0$ $\quad\therefore\quad b=-a \quad\cdots$ ①

$\therefore\quad \lim_{x\to -2}\dfrac{a\sqrt{x+3}+b}{x+2}=\lim_{x\to -2}\dfrac{a\sqrt{x+3}-a}{x+2}=\lim_{x\to -2}\dfrac{a\cancel{(x+2)}}{\cancel{(x+2)}\left(\sqrt{x+3}+1\right)}$

$=\lim_{x\to -2}\dfrac{a}{\sqrt{x+3}+1}=1 \quad\therefore\quad \dfrac{a}{2}=1 \quad\therefore\quad a=2$ 　これを ① に代入して $b=-2$

以上より $a=2,\ b=-2$

(4) 与式が成り立つには $\lim_{x\to 1}\left(a\sqrt{x+3}+b\right)=0$ $\quad\therefore\quad 2a+b=0$ $\quad\therefore\quad b=-2a \quad\cdots$ ①

$\therefore\quad \lim_{x\to 1}\dfrac{x^2+ax+b}{x-1}=\lim_{x\to 1}\dfrac{a\sqrt{x+3}-2a}{x-1}=\lim_{x\to 1}\dfrac{a\cancel{(x-1)}}{\cancel{(x-1)}\left(\sqrt{x+3}+2\right)}=\lim_{x\to 1}\dfrac{a}{\left(\sqrt{x+3}+2\right)}=1$

$\therefore\quad \dfrac{a}{4}=1 \quad\therefore\quad a=4$ 　これを ① に代入して $b=-8$

以上より $a=4,\ b=-8$

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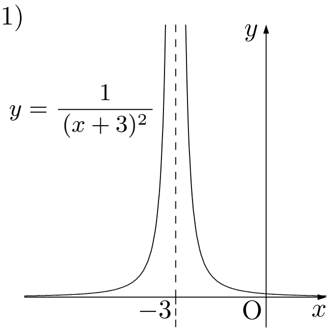
(1) グラフより ∞

(2) グラフより $-\infty$

(3) グラフより ∞

グラフは下図の通り

(1)



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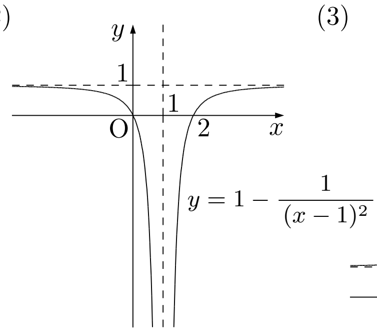

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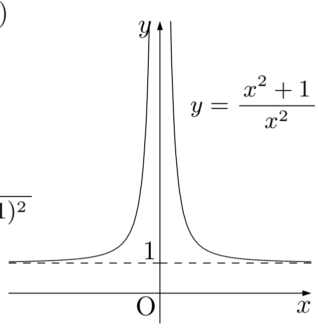

8

(1) $\lim_{x\to 1-0}\dfrac{|x-1|}{x-1}=\lim_{x\to 1-0}\dfrac{-\cancel{(x-1)}}{\cancel{x-1}}=\lim_{x\to 1-0}(-1)=-1$

(2) $\lim_{x\to 1+0}\dfrac{|1-x|}{x-1}=\lim_{x\to 1+0}\dfrac{\cancel{x-1}}{\cancel{x-1}}=\lim_{x\to 1+0}1=1$

(3) $\lim_{x\to -0}\dfrac{\sqrt{x^2}}{x}=\lim_{x\to -0}\dfrac{|\,x\,|}{x}=\lim_{x\to -0}\dfrac{-\cancel{x}}{\cancel{x}}=\lim_{x\to -0}(-1)=-1$

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(1) $\lim_{x\to\infty}\dfrac{1}{x-1}=0$

(2) $\lim_{x\to -\infty}\dfrac{1}{x^2-1}=0$

(3) $\lim_{x\to\infty}\left(1-\dfrac{1}{x^3}\right)=1$

(4) $\lim_{x\to\infty}(x^3-x)=\lim_{x\to\infty}x^3\left(1-\dfrac{1}{x^2}\right)=\infty$

(5) $\lim_{x\to -\infty}(x^3-x^2-x)=\lim_{x\to -\infty}x^3\left(1-\dfrac{1}{x}-\dfrac{1}{x^2}\right)=-\infty$

(6) $\lim_{x\to -\infty}\left(x^2-\dfrac{1}{x}\right)=\lim_{x\to -\infty}x^2\left(1-\dfrac{1}{x^3}\right)=\infty$

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(1) $\displaystyle \lim_{x\to\infty} \frac{1-x^2}{1+x^2} = \lim_{x\to\infty} \frac{\frac{1}{x^2}-1}{\frac{1}{x^2}+1} = -1$

(2) $\displaystyle \lim_{x\to-\infty} \frac{8x^3+1}{x^3+x+1} = \lim_{x\to-\infty} \frac{8+\frac{1}{x^3}}{1+\frac{1}{x^2}+\frac{1}{x^3}} = 8$

(3) $\displaystyle \lim_{x\to\infty} \frac{x^3-1}{x^2+1} = \lim_{x\to\infty} \frac{(x^2+1)x-(x+1)}{x^2+1} = \lim_{x\to\infty}\left(x-\frac{x+1}{x^2+1}\right) = \lim_{x\to\infty}\left(x-\frac{1+\frac{1}{x}}{x+\frac{1}{x}}\right) = \infty$

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(1) $\displaystyle \lim_{x\to\infty}\left(\frac{1}{3}\right)^x = 0$

(2) $\displaystyle \lim_{x\to\infty} \frac{2^x-3^x}{3^x} = \lim_{x\to\infty}\left\{\left(\frac{2}{3}\right)^x-1\right\} = -1$

(3) $\displaystyle \lim_{x\to-\infty} \frac{2^x+3^x}{4^x} = \lim_{x\to-\infty}\left\{\left(\frac{1}{2}\right)^x+\left(\frac{3}{4}\right)^x\right\} = \infty$

(4) $\displaystyle \lim_{x\to-\infty} \frac{5^x}{4^x+5^x} = \lim_{x\to-\infty} \frac{1}{\left(\frac{4}{5}\right)^x+1} = 0$

(5) $\displaystyle \lim_{x\to\infty} \frac{3^x+4^x}{3^x-4^x} = \lim_{x\to\infty} \frac{\left(\frac{3}{4}\right)^x+1}{\left(\frac{3}{4}\right)^x-1} = -1$

(6) $\displaystyle \lim_{x\to\infty} \frac{2^{-x}}{1+2^{-x}} = 0$

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(1) $\displaystyle \lim_{x\to\infty} \log_{\frac{1}{2}} x = -\infty$

(2) $\displaystyle \lim_{x\to\infty} \log_{0.1} \frac{1}{x} = \lim_{x\to\infty} \log_{0.1} x^{-1} = -\lim_{x\to\infty} \log_{0.1} x = \infty$

(3) $\displaystyle \lim_{x\to+0} \log_3 x = -\infty$

13

(1) $\displaystyle \lim_{x\to0} \frac{\sin 3x}{2x} = \lim_{x\to0}\left(\frac{3}{2}\cdot\frac{\sin 3x}{3x}\right) = \frac{3}{2}$

(2) $\displaystyle \lim_{x\to0} \frac{\tan 3x}{x} = \lim_{x\to0} \frac{\sin 3x}{x\cos 3x} = \lim_{x\to0}\left(\frac{\sin 3x}{3x}\cdot\frac{3}{\cos 3x}\right) = 3$

(3) $\displaystyle \lim_{x\to0} \frac{\sin 4x}{\sin 2x} = \lim_{x\to0} \frac{\frac{\sin 4x}{4x}}{\frac{1}{2}\cdot\frac{\sin 2x}{2x}} = 2$

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(1) $\displaystyle \lim_{x\to0} f(x) = \lim_{x\to0}(x^2-1) = -1 \neq 1 = f(0)$ $\quad\therefore$ 連続でない

(2) $\displaystyle \lim_{x\to0} f(x) = \lim_{x\to0} \frac{x^2+x}{x^2-x} = \lim_{x\to0} \frac{x+1}{x-1} = -1 = f(0)$ $\quad\therefore$ 連続である

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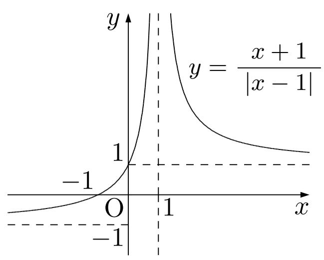
(1) $\displaystyle \lim_{x\to\infty} \frac{x^2+2x-3}{2-x} = \lim_{x\to\infty} \frac{(2-x)(-x-4)+5}{2-x} = \lim_{x\to\infty}\left(-x-4+\frac{5}{2-x}\right) = -\infty$

(2) $x>1$ のとき，$\displaystyle \frac{x+1}{|x-1|} = \frac{x+1}{x-1} = 1+\frac{2}{x-1}$

$x<1$ のとき，$\displaystyle \frac{x+1}{|x-1|} = -\frac{x+1}{x-1} = -1-\frac{2}{x-1}$

図より $\displaystyle \lim_{x\to1} \frac{x+1}{|x-1|} = \infty$



(3) $t = \dfrac{1}{x}$ とおくと，$x \longrightarrow +0$ のとき $t \longrightarrow \infty$ だから

$$\lim_{x \to +0} \left(\frac{1}{2}\right)^{\frac{1}{x}} = \lim_{t \to \infty} \left(\frac{1}{2}\right)^t = 0$$

(4) $\displaystyle\lim_{x \to \frac{\pi}{2}-0} \tan x = \infty$

(5) $\displaystyle\lim_{x \to \pi} \frac{\sin 2x}{\tan x} = \lim_{x \to \pi} \frac{2\cancel{\sin x}\cos x}{\dfrac{\cancel{\sin x}}{\cos x}} = \lim_{x \to \pi} 2\cos^2 x = 2$

(6) $\displaystyle\lim_{x \to \infty} \mathrm{Tan}^{-1}\, x = \frac{\pi}{2}$

(7) $\displaystyle\lim_{x \to +0} \log_2 |x| = \lim_{x \to +0} \log_2 x = -\infty$

また，$t = -x$ とおくと，$x \longrightarrow -0$ のとき $t \longrightarrow +0$ だから

$$\lim_{x \to -0} \log_2 |x| = \lim_{x \to -0} \log_2(-x) = \lim_{t \to +0} \log_2 t = -\infty$$

$\therefore \displaystyle\lim_{x \to 0} \log_2 |x| = -\infty$

(8) $\displaystyle\lim_{x \to 0}\left\{ \log_3(x^2 + x) - \log_3 x \right\} = \lim_{x \to 0} \log_3 \frac{x^2 + x}{x} = \lim_{x \to 0} \log_3(x + 1) = 0$

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(1) $\displaystyle\lim_{x \to 2} \frac{2x^2 - 3x - 2}{3x^2 - 7x + 2} = \lim_{x \to 2} \frac{(2x + 1)\cancel{(x - 2)}}{(3x - 1)\cancel{(x - 2)}} = \lim_{x \to 2} \frac{2x + 1}{3x - 1} = 1$

(2) $\displaystyle\lim_{x \to 0} \frac{\sin 4x \tan 3x}{2x^2} = \lim_{x \to 0} \left(6 \cdot \frac{\sin 4x}{4x} \cdot \frac{\sin 3x}{3x} \cdot \frac{1}{\cos 3x}\right) = 6$

(3) $\displaystyle\lim_{x \to 0} \frac{1 - \cos 2x}{x \sin x} = \lim_{x \to 0} \left(\frac{2}{x \sin x} \cdot \frac{1 - \cos 2x}{2}\right) = \lim_{x \to 0} \left(\frac{2}{x \sin x} \cdot \sin^2 x\right) = \lim_{x \to 0} \left(2 \cdot \frac{\sin x}{x}\right) = 2$

(4) $t = \dfrac{1}{x}$ とおくと，$x \longrightarrow \infty$ のとき $t \longrightarrow +0$ だから

$$\lim_{x \to \infty} x \sin \frac{1}{x} = \lim_{t \to +0} \frac{\sin t}{t} = 1$$

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(1) $\displaystyle\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} = \lim_{x \to 4} \frac{\cancel{(x - 4)}\left(\sqrt{x} + 2\right)}{\cancel{x - 4}} = \lim_{x \to 4}\left(\sqrt{x} + 2\right) = 4$

(2) $\displaystyle\lim_{x \to 3} \frac{\sqrt{x + 6} - 3}{\sqrt{x + 1} - 2} = \lim_{x \to 3} \frac{\cancel{(x - 3)}\left(\sqrt{x + 1} + 2\right)}{\cancel{(x - 3)}\left(\sqrt{x + 6} + 3\right)} = \lim_{x \to 3} \frac{\sqrt{x + 1} + 2}{\sqrt{x + 6} + 3} = \frac{2}{3}$

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(1) $\displaystyle\lim_{x \to \infty} \left(x - \sqrt{x^2 - 2x}\,\right) = \lim_{x \to \infty} \frac{\cancel{x^2} - (\cancel{x^2} - 2x)}{x + \sqrt{x^2 - 2x}} = \lim_{x \to \infty} \frac{2x}{x + \sqrt{x^2 - 2x}} = \lim_{x \to \infty} \frac{2}{1 + \sqrt{1 - \dfrac{2}{x}}} = 1$

(2) $\displaystyle\lim_{x \to \infty} \left\{ \log_2(2x + 1) - \log_2 x \right\} = \lim_{x \to \infty} \log_2 \frac{2x + 1}{x} = \lim_{x \to \infty} \log_2 \left(2 + \frac{1}{x}\right) = 1$

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(1) $\displaystyle\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x^2 + x}{x} = \lim_{x \to 0}(x + 1) = 1 \neq 0 = f(0)$ 　　$\therefore$ 連続でない

(2) $\displaystyle\lim_{x \to +0} f(x) = \lim_{x \to 0} \frac{\sin x}{x} = 1$ 　　$\displaystyle\lim_{x \to -0} f(x) = \lim_{x \to 0} \frac{\sin x}{-x} = -1$

$\displaystyle\lim_{x \to +0} f(x) \neq \lim_{x \to -0} f(x)$ 　より、$\displaystyle\lim_{x \to 0} f(x)$ は存在しない 　　$\therefore$ 連続でない

(3) $t = -\dfrac{1}{x^2}$ とおくと，$x \longrightarrow 0$ のとき $t \longrightarrow -\infty$ だから

$\displaystyle\lim_{x \to 0} f(x) = \lim_{x \to 0} 2^{-\frac{1}{x^2}} = \lim_{t \to -\infty} 2^t = 0 = f(0)$ 　　$\therefore$ 連続である

(4) $t = \log_2 |x|$ とおくと，$x \longrightarrow 0$ のとき $t \longrightarrow -\infty$ だから

$\displaystyle\lim_{x \to 0} f(x) = \lim_{x \to 0} 2^{\log_2 |x|} = \lim_{x \to -\infty} 2^t = 0 = f(0)$ 　　$\therefore$ 連続である

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(1) 任意の $x \neq 0$ に対し，$-1 \leqq \cos \dfrac{1}{x} \leqq 1$ 　　$\therefore$ 　$-|x| \leqq \cos \dfrac{1}{x} \leqq |x|$

また，$\displaystyle\lim_{x \to 0}(-|x|) = \lim_{x \to 0} |x| = 0$ 　だから，はさみうちの原理より，$\displaystyle\lim_{x \to 0} x \cos \frac{1}{x} = 0$

(2) 任意の $x \neq 0$ に対し，$-1 \leqq \sin x \leqq 1$ $\quad\therefore\quad -\dfrac{1}{|x|} \leqq \dfrac{\sin x}{x} \leqq \dfrac{1}{|x|}$

また，$\displaystyle\lim_{x\to\infty}\left(-\frac{1}{|x|}\right) = \lim_{x\to\infty}\left(\frac{1}{|x|}\right) = 0$ だから，はさみうちの原理より，$\displaystyle\lim_{x\to\infty}\frac{\sin x}{x} = 0$

(3) 任意の $x \neq 0$ に対し，$-1 \leqq \sin 2x \leqq 1$ $\quad\therefore\quad -\dfrac{1}{|x|} \leqq \dfrac{\sin 2x}{x} \leqq \dfrac{1}{|x|}$

また，$\displaystyle\lim_{x\to-\infty}\left(-\frac{1}{|x|}\right) = \lim_{x\to-\infty}\left(\frac{1}{|x|}\right) = 0$ だから，はさみうちの原理より，$\displaystyle\lim_{x\to-\infty}\frac{\sin 2x}{x} = 0$