

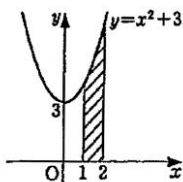
2節 積分法の応用

A 問題

113

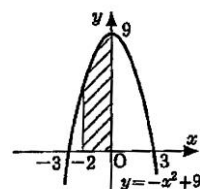
- (1) 求める面積は右図の斜線部分だから

$$\begin{aligned} S &= \int_1^2 (x^2 + 3) dx \\ &= \left[\frac{1}{3} x^3 + 3x \right]_1^2 \\ &= \left(\frac{8}{3} + 6 \right) - \left(\frac{1}{3} + 3 \right) \\ &= \frac{7}{3} + 3 = \frac{16}{3} \end{aligned}$$



- (2) 求める面積は右図の斜線部分だから

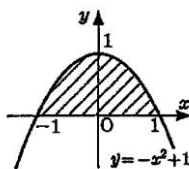
$$\begin{aligned} S &= \int_{-2}^0 (-x^2 + 9) dx \\ &= \left[-\frac{1}{3} x^3 + 9x \right]_{-2}^0 \\ &= 0 - \left(\frac{8}{3} - 18 \right) = \frac{46}{3} \end{aligned}$$



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- (1) 求める面積は右図の斜線部分だから

$$\begin{aligned} S &= \int_{-1}^1 (-x^2 + 1) dx \\ &= \left[-\frac{1}{3} x^3 + x \right]_{-1}^1 \\ &= \left(-\frac{1}{3} + 1 \right) - \left(\frac{1}{3} - 1 \right) = \frac{4}{3} \end{aligned}$$



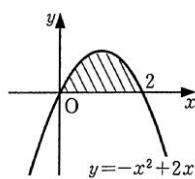
- (2) 放物線と x 軸との交点の x 座標は

$$-x^2 + 2x = 0 \text{ を解いて}$$

$$-x(x - 2) = 0 \quad \therefore x = 0, 2$$

求める面積は右図の斜線部分だから

$$\begin{aligned} S &= \int_0^2 (-x^2 + 2x) dx \\ &= \left[-\frac{1}{3} x^3 + x^2 \right]_0^2 \\ &= \left(-\frac{8}{3} + 4 \right) - 0 = \frac{4}{3} \end{aligned}$$

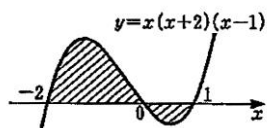


$$\begin{aligned} \text{(別解)} \quad S &= \int_0^2 (-x^2 + 2x) dx \\ &= -\int_0^2 x(x - 2) dx \\ &= \frac{(2 - 0)^3}{6} = \frac{4}{3} \end{aligned}$$

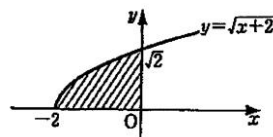
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- (1) $y = x(x + 2)(x - 1)$

$$\begin{aligned} S &= \int_{-2}^0 (x^3 + x^2 - 2x) dx \\ &= \int_0^1 (x^3 + x^2 - 2x) dx = \frac{37}{12} \end{aligned}$$

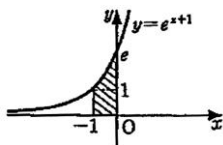


$$\begin{aligned} (2) \quad S &= \int_{-2}^0 \sqrt{x + 2} dx = \left[\frac{2}{3} (x + 2)^{\frac{3}{2}} \right]_{-2}^0 \\ &= \frac{4\sqrt{2}}{3} \end{aligned}$$



$$(3) \quad S = \int_{-1}^0 e^{x+1} dx$$

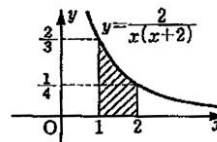
$$= \left[e^{x+1} \right]_{-1}^0 = e - 1$$



$$(4) \quad 1 \leq x \leq 2 \quad \text{において} \quad y = \frac{2}{x(x+2)} > 0 \quad \text{より}$$

$$S = \int_1^2 \frac{2}{x(x+2)} dx = \int_1^2 \left(\frac{1}{x} - \frac{1}{x+2} \right) dx$$

$$= \left[\log x - \log(x+2) \right]_1^2 = \log \frac{3}{2}$$



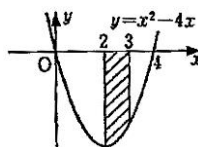
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- (1) 求める面積は右図の斜線部分だから
求める面積を S とすると

$$S = -\int_2^3 (x^2 - 4x) dx = -\left[\frac{1}{3}x^3 - 2x^2 \right]_2^3$$

$$= -\left\{ (9 - 18) - \left(\frac{8}{3} - 8 \right) \right\}$$

$$= -\left(-1 - \frac{8}{3} \right) = \frac{11}{3}$$



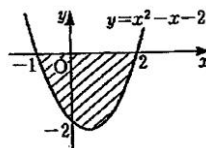
- (2) 放物線と x 軸との交点の x 座標は
 $x^2 - x - 2 = 0$ を解いて
 $(x+1)(x-2) = 0$
 $\therefore x = -1, 2$

求める面積は右図の斜線部分だから

$$S = -\int_{-1}^2 (x^2 - x - 2) dx$$

$$= -\left[\frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x \right]_{-1}^2$$

$$= -\left\{ \frac{1}{3}(8+1) - \frac{1}{2}(4-1) - 2(2+1) \right\} = \frac{9}{2}$$



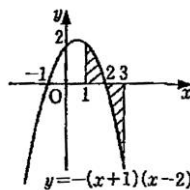
$$(別解) \quad S = -\int_{-1}^2 (x^2 - x - 2) dx$$

$$= -\int_{-1}^2 (x+1)(x-2) dx$$

$$= \frac{\{2 - (-1)\}^3}{6} = \frac{9}{2}$$

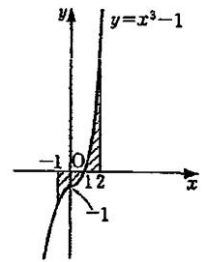
(1) 求める面積は下図の斜線部分だから

$$\begin{aligned}
 S &= \int_1^2 \{-(x+1)(x-2)\} dx \\
 &\quad + \left[-\int_2^3 \{-(x+1)(x-2)\} dx \right] \\
 &= \int_1^2 (-x^2 + x + 2) dx + \int_2^3 (x^2 - x - 2) dx \\
 &= \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x \right]_1^2 + \left[\frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x \right]_2^3 \\
 &= -\frac{1}{3}(8-1) + \frac{1}{2}(4-1) + 2(2-1) \\
 &\quad + \frac{1}{3}(27-8) - \frac{1}{2}(9-4) - 2(3-2) \\
 &= -\frac{7}{3} + \frac{3}{2} + 2 + \frac{19}{3} - \frac{5}{2} - 2 = 3
 \end{aligned}$$



(2) 求める面積は下図の斜線部分だから

$$\begin{aligned}
 S &= -\int_{-1}^1 (x^3 - 1) dx + \int_1^2 (x^3 - 1) dx \\
 &= -2 \int_0^1 (-1) dx + \left[\frac{1}{4}x^4 - x \right]_1^2 \\
 &= 2 \left[x \right]_0^1 + (4-2) - \left(\frac{1}{4} - 1 \right) \\
 &= 2(1-0) + 3 - \frac{1}{4} \\
 &= 5 - \frac{1}{4} = \frac{19}{4}
 \end{aligned}$$

(1) 交点の x 座標は

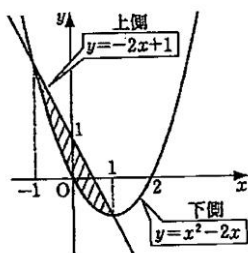
$$\begin{aligned}
 x^2 - 2x &= -2x + 1 \text{ を解いて } x^2 - 1 = 0 \\
 (x+1)(x-1) &= 0 \quad \therefore x = \pm 1
 \end{aligned}$$

求める面積は下図の斜線部分だから

$$\begin{aligned}
 S &= \int_{-1}^1 \{(-2x+1) - (x^2-2x)\} dx \\
 &= \int_{-1}^1 (-x^2 + 1) dx \\
 &= \left[-\frac{1}{3}x^3 + x \right]_{-1}^1 \\
 &= \left(-\frac{1}{3} + 1 \right) - \left(\frac{1}{3} - 1 \right) = \frac{4}{3}
 \end{aligned}$$

$$(\text{別解}) \quad S = \int_{-1}^1 (-x^2 + 1) dx$$

$$\begin{aligned}
 &= \int_{-1}^1 (x+1)(x-1) dx \\
 &= \frac{\{1 - (-1)\}^3}{6} = \frac{4}{3}
 \end{aligned}$$

(2) 交点の x 座標は

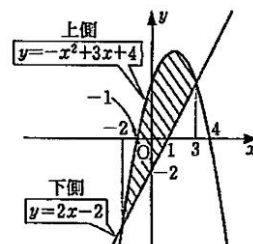
$$\begin{aligned}
 -x^2 + 3x + 4 &= 2x - 2 \text{ を解いて } x^2 - x - 6 = 0 \\
 (x+2)(x-3) &= 0 \quad \therefore x = -2, 3
 \end{aligned}$$

求める面積は下図の斜線部分だから

$$\begin{aligned}
 S &= \int_{-2}^3 \{(-x^2 + 3x + 4) - (2x - 2)\} dx \\
 &= \int_{-2}^3 (-x^2 + x + 6) dx \\
 &= \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 6x \right]_{-2}^3 \\
 &= \left(-9 + \frac{9}{2} + 18 \right) - \left(\frac{8}{3} + 2 - 12 \right) = \frac{125}{6}
 \end{aligned}$$

$$(\text{別解}) \quad S = \int_{-2}^3 (-x^2 + x + 6) dx$$

$$\begin{aligned}
 &= -\int_{-2}^3 (x+2)(x-3) dx \\
 &= \frac{\{3 - (-2)\}^3}{6} = \frac{125}{6}
 \end{aligned}$$



- (3) 2 曲線の交点の x 座標は

$$x^2 - 1 = -x^2 - 2x - 1 \text{ を解いて}$$

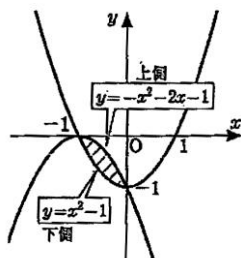
$$2x^2 + 2x = 0$$

$$2x(x+1) = 0 \quad \therefore x = 0, -1$$

求める面積は下図の斜線部分だから

$$\begin{aligned} S &= \int_{-1}^0 \{(-x^2 - 2x - 1) - (x^2 - 1)\} dx \\ &= \int_{-1}^0 (-2x^2 - 2x) dx \\ &= \left[-\frac{2}{3}x^3 - x^2 \right]_{-1}^0 \\ &= 0 - \left(\frac{2}{3} - 1 \right) = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \text{(別解)} \quad S &= \int_{-1}^0 (-2x^2 - 2x) dx \\ &= -2 \int_{-1}^0 (x^2 + x) dx \\ &= -2 \int_{-1}^0 (x+1)x dx \\ &= \frac{2(0+1)^3}{6} = \frac{1}{3} \end{aligned}$$



- (4) 2 曲線の交点の x 座標は

$$2x^2 + 2x - 3 = -x^2 + 2x \text{ を解いて}$$

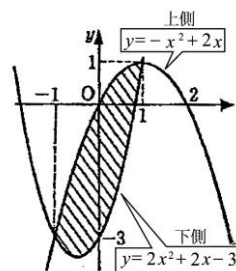
$$3x^2 - 3 = 0$$

$$3(x+1)(x-1) = 0 \quad \therefore x = \pm 1$$

求める面積は下図の斜線部分だから

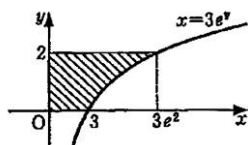
$$\begin{aligned} S &= \int_{-1}^1 \{(-x^2 + 2x) - (2x^2 + 2x - 3)\} dx \\ &= \int_{-1}^1 (-3x^2 + 3) dx \\ &= \left[-x^3 + 3x \right]_{-1}^1 \\ &= (-1 + 3) - (1 - 3) = 4 \end{aligned}$$

$$\begin{aligned} \text{(別解)} \quad S &= \int_{-1}^1 (-3x^2 + 3) dx \\ &= -3 \int_{-1}^1 (x+1)(x-1) dx \\ &= \frac{3(1+1)^3}{6} = 4 \end{aligned}$$



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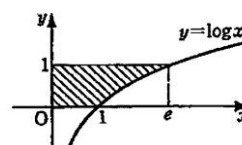
$$\begin{aligned} (1) \quad S &= \int_0^2 3e^y dy \\ &= \left[3e^y \right]_0^2 \\ &= 3e^2 - 3 \end{aligned}$$



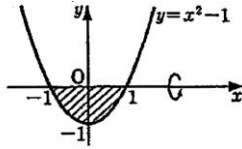
$$(2) \quad y = \log x \text{ より } x = e^y$$

$$S = \int_0^1 e^y dy = \left[e^y \right]_0^1 = e - 1$$

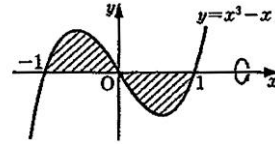
$$\begin{aligned} \text{(別解)} \quad S &= \int_0^e dx - \int_1^e \log x dx \\ &= \left[x \right]_0^e - \left[x \log x - x \right]_1^e = e - 1 \end{aligned}$$



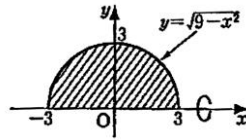
$$\begin{aligned}
 (1) \quad V &= \pi \int_{-1}^1 (x^2 - 1)^2 dx \\
 &= 2\pi \int_0^1 (x^4 - 2x^2 + 1) dx \\
 &= 2\pi \left[\frac{x^5}{5} - \frac{2}{3}x^3 + x \right]_0^1 = \frac{16}{15}\pi
 \end{aligned}$$



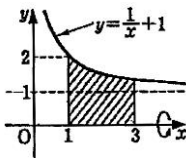
$$\begin{aligned}
 (2) \quad V &= \pi \int_{-1}^1 (x^3 - x)^2 dx \\
 &= 2\pi \int_0^1 (x^6 - 2x^4 + x^2) dx \\
 &= 2\pi \left[\frac{x^7}{7} - \frac{2}{5}x^5 + \frac{x^3}{3} \right]_0^1 = \frac{16}{105}\pi
 \end{aligned}$$



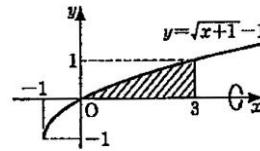
$$\begin{aligned}
 (3) \quad V &= \pi \int_{-3}^3 (\sqrt{9 - x^2})^2 dx \\
 &= 2\pi \int_0^3 (9 - x^2) dx \\
 &= 2\pi \left[9x - \frac{x^3}{3} \right]_0^3 = 36\pi
 \end{aligned}$$



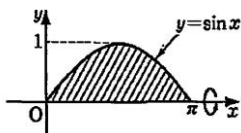
$$\begin{aligned}
 (1) \quad V &= \pi \int_1^3 \left(\frac{1}{x} + 1 \right)^2 dx \\
 &= \pi \int_1^3 \left(\frac{1}{x^2} + \frac{2}{x} + 1 \right) dx \\
 &= \pi \left[-\frac{1}{x} + 2 \log x + x \right]_1^3 = \frac{8 + 6 \log 3}{3} \pi
 \end{aligned}$$



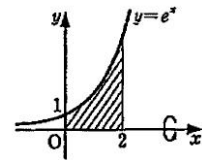
$$\begin{aligned}
 (2) \quad V &= \pi \int_0^3 (\sqrt{x+1} - 1)^2 dx \\
 &= \pi \int_0^3 (x + 2 - 2\sqrt{x+1}) dx \\
 &= \pi \left[\frac{x^2}{2} + 2x - \frac{4}{3}(x+1)^{3/2} \right]_0^3 = \frac{7}{6}\pi
 \end{aligned}$$



$$\begin{aligned}
 (3) \quad V &= \pi \int_0^\pi \sin^2 x dx \\
 &= \pi \int_0^\pi \frac{1 - \cos 2x}{2} dx \\
 &= \pi \left[\frac{1}{2}x - \frac{1}{4}\sin 2x \right]_0^\pi = \frac{1}{2}\pi^2
 \end{aligned}$$

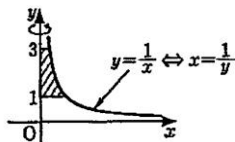


$$\begin{aligned}
 (4) \quad V &= \pi \int_0^2 e^{2x} dx \\
 &= \pi \left[\frac{1}{2}e^{2x} \right]_0^2 \\
 &= \frac{e^4 - 1}{2} \pi
 \end{aligned}$$



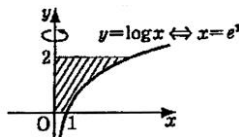
$$(1) \quad V = \pi \int_1^3 x^2 dy = \pi \int_1^3 \frac{1}{y^2} dy$$

$$= \pi \left[-\frac{1}{y} \right]_1^3 = \frac{2}{3} \pi$$



$$(2) \quad V = \pi \int_0^2 x^2 dy = \pi \int_0^2 e^{2y} dy$$

$$= \pi \left[\frac{1}{2} e^{2y} \right]_0^2 = \frac{e^4 - 1}{2} \pi$$



B 問題

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$$(1) \quad \cos x = \sin \frac{1}{2} x \quad \text{より}$$

$$2 \sin^2 \frac{1}{2} x + \sin \frac{1}{2} x - 1 = 0$$

$$\left(2 \sin \frac{1}{2} x - 1 \right) \left(\sin \frac{1}{2} x + 1 \right) = 0$$

$$\therefore \sin \frac{1}{2} x = \frac{1}{2}, -1$$

$$0 \leq x \leq 2\pi \quad \text{より} \quad x = \frac{\pi}{3}, \frac{5}{3}\pi$$

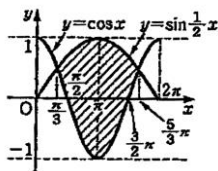
$$\text{また, } \frac{\pi}{3} \leq x \leq \frac{5}{3}\pi \quad \text{で} \quad \sin \frac{1}{2} x \geq \cos x \quad \text{である。}$$

$$\text{よって} \quad S = \int_{\frac{\pi}{3}}^{\frac{5}{3}\pi} \left(\sin \frac{1}{2} x - \cos x \right) dx$$

$$= \left[-2 \cos \frac{1}{2} x - \sin x \right]_{\frac{\pi}{3}}^{\frac{5}{3}\pi} = 3\sqrt{3}$$

(別解) グラフの対称性から

$$S = 2 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\sin \frac{1}{2} x - \cos x \right) dx = 3\sqrt{3}$$



$$(2) \quad \sin 2x = \sin x \quad \text{より} \quad \sin x (2 \cos x - 1) = 0$$

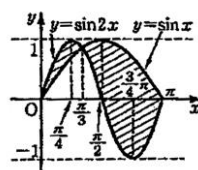
$$\sin x = 0, \quad \cos x = \frac{1}{2}$$

$$0 \leq x \leq \pi \quad \text{より} \quad x = 0, \frac{\pi}{3}, \pi \quad \text{よって}$$

$$S = \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x) dx + \int_{\frac{\pi}{3}}^{\pi} (\sin x - \sin 2x) dx$$

$$= \left[-\frac{1}{2} \cos 2x + \cos x \right]_0^{\frac{\pi}{3}} + \left[-\cos x + \frac{1}{2} \cos 2x \right]_{\frac{\pi}{3}}^{\pi}$$

$$= \frac{5}{2}$$



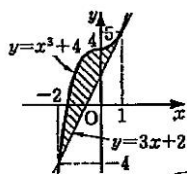
$$124 \quad y' = 3x^2 \quad \text{より, 接線の方程式は} \quad y = 3x + 2$$

$$x^3 + 4 = 3x + 2 \quad \text{より, 共有点の} x \text{ 座標は}$$

$$x = 1, -2$$

$$\text{よって} \quad S = \int_{-2}^1 \left\{ (x^3 + 4) - (3x + 2) \right\} dx$$

$$= \left[\frac{x^4}{4} - \frac{3}{2} x^2 + 2x \right]_{-2}^1 = \frac{27}{4}$$

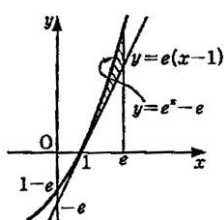


(1) $y = e^x - e$ より $y' = e^x$

$x = 1$ のとき $y' = e$ となるから

接線の方程式は $y = e(x - 1)$

$$\begin{aligned} S &= \int_1^e \{ (e^x - e) - e(x - 1) \} dx \\ &= \int_1^e (e^x - ex) dx = \left[e^x - \frac{1}{2} ex^2 \right]_1^e \\ &= \left(e^e - \frac{1}{2} e^3 \right) - \left(e - \frac{1}{2} e \right) \\ &= e^e - \frac{1}{2} e^3 - \frac{1}{2} e \end{aligned}$$

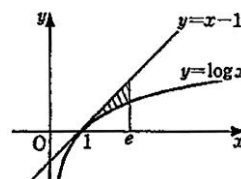


(2) $y = \log x$ より $y' = \frac{1}{x}$

$x = 1$ のとき $y' = 1$ となるから

接線の方程式は $y = x - 1$

$$\begin{aligned} S &= \int_1^e \{ (x - 1) - \log x \} dx \\ &= \left[\frac{1}{2} x^2 - x \right]_1^e - \left[x \log x - x \right]_1^e \\ &= \left(\frac{1}{2} e^2 - e \right) - \left(\frac{1}{2} - 1 \right) - (e \log e - e + 1) \\ &= \frac{1}{2} e^2 - e - \frac{1}{2} \end{aligned}$$

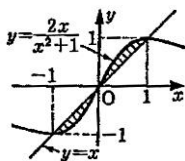


(1) グラフはともに原点に関して対称である。

$$\frac{2x}{x^2 + 1} = x \text{ より } x(x + 1)(x - 1) = 0$$

$$\therefore x = 0, \pm 1$$

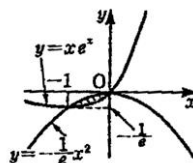
$$\begin{aligned} S &= 2 \int_0^1 \left(\frac{2x}{x^2 + 1} - x \right) dx \\ &= 2 \left[\log(x^2 + 1) - \frac{1}{2} x^2 \right]_0^1 = 2 \log 2 - 1 \end{aligned}$$



(2) $xe^x = -\frac{1}{e} x^2$ より $x(e^{x+1} + x) = 0$

$$\therefore x = 0, -1$$

$$\begin{aligned} S &= \int_{-1}^0 \left(-\frac{1}{e} x^2 - xe^x \right) dx \\ &= \left[-\frac{1}{3e} x^3 \right]_{-1}^0 - \left[xe^x \right]_{-1}^0 + \int_{-1}^0 e^x dx \\ &= -\frac{1}{3e} - \frac{1}{e} + \left[e^x \right]_{-1}^0 = 1 - \frac{7}{3e} \end{aligned}$$



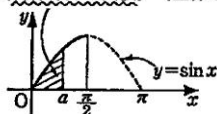
127 $2 \int_0^a \sin x dx = \int_0^{\frac{\pi}{2}} \sin x dx$

$$2 \left[-\cos x \right]_0^a = \left[-\cos x \right]_0^{\frac{\pi}{2}}$$

$$-2 \cos a + 2 = 1 \quad \cos a = \frac{1}{2}$$

$$0 < a < \frac{\pi}{2} \text{ より } a = \frac{\pi}{3}$$

2 × (斜線部分の面積) = (全体の面積)



切り口の断面積を $S(x)$ とおくと

$$S(x) = \frac{1}{2} \cdot \sin x \cdot \sin x \cdot \sin 60^\circ = \frac{\sqrt{3}}{4} \sin^2 x$$

$$\begin{aligned} \text{よって } V &= \int_0^\pi \frac{\sqrt{3}}{4} \sin^2 x \, dx \\ &= \frac{\sqrt{3}}{4} \int_0^\pi \frac{1 - \cos 2x}{2} \, dx \\ &= \frac{\sqrt{3}}{8} \left[x - \frac{1}{2} \sin 2x \right]_0^\pi = \frac{\sqrt{3}}{8} \pi \end{aligned}$$

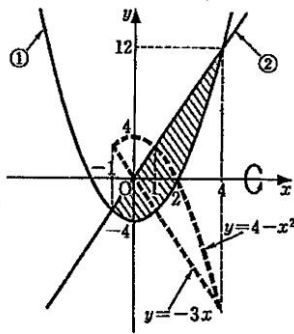
- (1) $y = x^2 - 4 \cdots \textcircled{1}$, $y = 3x \cdots \textcircled{2}$ のグラフは
下のようになり, $\textcircled{1}$, $\textcircled{2}$ の $y < 0$ の部分を
 x 軸に対称に折り返して考える。

$$x^2 - 4 = 3x \text{ より } (x+1)(x-4) = 0$$

$$\therefore x = -1, 4$$

また, $y = x^2 - 4$ は y 軸に関して対称だから

$$\begin{aligned} V &= 2\pi \int_0^1 (4 - x^2)^2 \, dx - \pi \int_{-1}^0 (-3x)^2 \, dx \\ &\quad + \pi \int_1^4 (3x)^2 \, dx - \pi \int_2^4 (x^2 - 4)^2 \, dx \\ &= 2\pi \left[\frac{x^5}{5} - \frac{8}{3}x^3 + 16x \right]_0^1 - \pi \left[3x^3 \right]_{-1}^0 \\ &\quad + \pi \left[3x^3 \right]_1^4 - \pi \left[\frac{x^5}{5} - \frac{8}{3}x^3 + 16x \right]_2^4 = 132\pi \end{aligned}$$



- (2) $y = \sin x \cdots \textcircled{1}$, $y = \sin 2x \cdots \textcircled{2}$ のグラフは
下のようになり, $\textcircled{2}$ の $y < 0$ の部分を
 x 軸に対称に折り返して考える。

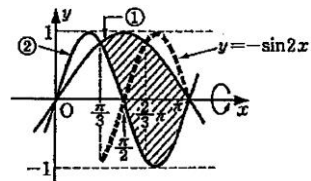
$\textcircled{1}$, $\textcircled{2}$ の交点は

$$\sin x = \sin 2x \text{ より } \sin x (2 \cos x - 1) = 0$$

$$\sin x = 0, \cos x = \frac{1}{2} \quad \frac{\pi}{3} \leq x \leq \pi \text{ より}$$

$$x = \frac{\pi}{3}, \pi$$

$$\begin{aligned} V &= \pi \int_{\frac{\pi}{3}}^{\frac{2}{3}\pi} \sin^2 x \, dx + \pi \int_{\frac{2}{3}\pi}^{\pi} \sin^2 2x \, dx \\ &\quad - \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^2 2x \, dx \\ &= \frac{\pi}{2} \int_{\frac{\pi}{3}}^{\frac{2}{3}\pi} (1 - \cos 2x) \, dx + \frac{\pi}{2} \int_{\frac{2}{3}\pi}^{\pi} (1 - \cos 4x) \, dx \\ &\quad - \frac{\pi}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1 - \cos 4x) \, dx \\ &= \frac{\pi}{2} \left\{ \left[x - \frac{1}{2} \sin 2x \right]_{\frac{\pi}{3}}^{\frac{2}{3}\pi} + \left[x - \frac{1}{4} \sin 4x \right]_{\frac{2}{3}\pi}^{\pi} \right. \\ &\quad \left. - \left[x - \frac{1}{4} \sin 4x \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \right\} = \frac{2\pi^2 + 3\sqrt{3}\pi}{8} \end{aligned}$$



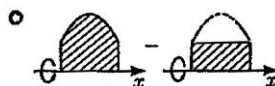
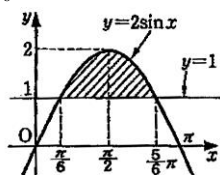
(1) $2 \sin x = 1$ とすると $\sin x = \frac{1}{2}$

$0 \leq x \leq \pi$ より $x = \frac{\pi}{6}, \frac{5}{6}\pi$

グラフは直線 $x = \frac{\pi}{2}$ に関して対称

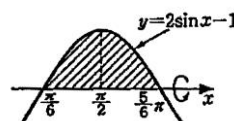
であるから、求める体積 V は

$$\begin{aligned} V &= 2\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2 \sin x)^2 dx - 2\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 1^2 dx \\ &= 8\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} dx - 2\pi \left[x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \\ &= 8\pi \left[\frac{1}{2} x - \frac{1}{4} \sin 2x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} - \frac{2}{3} \pi^2 \\ &= \frac{(2\pi + 3\sqrt{3})\pi}{3} \end{aligned}$$



(2) $V = 2\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2 \sin x - 1)^2 dx$

$$\begin{aligned} &= 2\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (4 \sin^2 x - 4 \sin x + 1) dx \\ &= 2\pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \{ 2(1 - \cos 2x) - 4 \sin x + 1 \} dx \\ &= 2\pi \left[-\sin 2x + 4 \cos x + 3x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \\ &= (2\pi - 3\sqrt{3})\pi \end{aligned}$$



131 $y = x^2 \dots \textcircled{1}$ とすると $y' = 2x$

$\textcircled{1}$ 上の点 (t, t^2) における接線の方程式は $y - t^2 = 2t(x - t)$

$\therefore y = 2tx - t^2 \dots \textcircled{2}$

これが点 $(1, -3)$ を通るためには

$-3 = 2t - t^2$

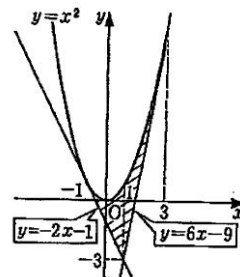
$t^2 - 2t - 3 = 0$

$(t+1)(t-3) = 0$

$\therefore t = -1, 3$

$t = -1$ のとき $\textcircled{2}$ から $y = -2x - 1$

$t = 3$ のとき $\textcircled{2}$ から $y = 6x - 9$



また求める面積は右図の斜線部分になるから

求める面積を S とすると

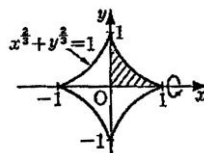
$$\begin{aligned} S &= \int_{-1}^1 \{ x^2 - (-2x - 1) \} dx + \int_1^3 \{ x^2 - (6x - 9) \} dx \\ &= \int_{-1}^1 (x^2 + 2x + 1) dx + \int_1^3 (x^2 - 6x + 9) dx \\ &= 2 \left[\frac{1}{3} x^3 + x \right]_0^1 + \left[\frac{1}{3} x^3 - 3x^2 + 9x \right]_1^3 \\ &= 2 \left\{ \left(\frac{1}{3} + 1 \right) - 0 \right\} + (9 - 27 + 27) - \left(\frac{1}{3} - 3 + 9 \right) \\ &= \frac{16}{3} \end{aligned}$$

発展問題

- 132 曲線 $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ はアステロイドであり、
 グラフは右図である。また

$$y^{\frac{2}{3}} = 1 - x^{\frac{2}{3}} \quad \text{より} \quad y^2 = \left(1 - x^{\frac{2}{3}}\right)^3$$

$$\begin{aligned} V &= \pi \int_0^1 \left(1 - x^{\frac{2}{3}}\right)^3 dx = \pi \int_0^1 \left(1 - 3x^{\frac{2}{3}} + 3x^{\frac{4}{3}} - x^2\right) dx \\ &= \pi \left[x - \frac{9}{5} x^{\frac{5}{3}} + \frac{9}{7} x^{\frac{7}{3}} - \frac{1}{3} x^3 \right]_0^1 = \frac{16}{105} \pi \end{aligned}$$



133

関数 $y = x^3$ と直線 $y = x$ の交点は

$A(1, 1)$, また $OA = \sqrt{2}$ である。

$0 \leq x \leq 1$ として, $y = x^3$ 上の点 $P(x, x^3)$ から
 直線 $y = x$ に垂線 PH を下ろし $PH = l$, $OH = t$
 とおくと、右図で $\triangle PHQ$ は二等辺三角形で

$$l = PH = \frac{PQ}{\sqrt{2}} = \frac{x - x^3}{\sqrt{2}}$$

また, $\triangle ORQ$ で $\frac{x}{t+l} = \cos 45^\circ = \frac{1}{\sqrt{2}}$ より

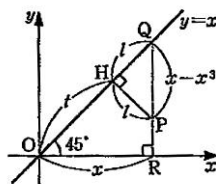
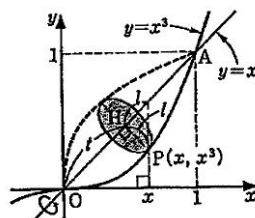
$$t = \sqrt{2}x - l = \sqrt{2}x - \frac{x - x^3}{\sqrt{2}} = \frac{x + x^3}{\sqrt{2}}$$

したがって $\frac{dt}{dx} = \frac{1+3x^2}{\sqrt{2}}$

$$dt = \frac{1+3x^2}{\sqrt{2}} dx \quad \begin{array}{|c|c|} \hline t & 0 \rightarrow \sqrt{2} \\ \hline x & 0 \rightarrow 1 \\ \hline \end{array}$$

よって $V = \int_0^{\sqrt{2}} \pi t^2 dt$

$$\begin{aligned} &= \pi \int_0^1 \left(\frac{x - x^3}{\sqrt{2}} \right)^2 \cdot \frac{1+3x^2}{\sqrt{2}} dx \\ &= \frac{\pi}{2\sqrt{2}} \int_0^1 (3x^8 - 5x^6 + x^4 + x^2) dx \\ &= \frac{\pi}{2\sqrt{2}} \left[\frac{1}{3} x^9 - \frac{5}{7} x^7 + \frac{1}{5} x^5 + \frac{1}{3} x^3 \right]_0^1 \\ &= \frac{\pi}{2\sqrt{2}} \left(\frac{1}{3} - \frac{5}{7} + \frac{1}{5} + \frac{1}{3} \right) = \frac{4\sqrt{2}}{105} \pi \end{aligned}$$



2 章の問題

1

$$\begin{aligned}
 (1) \quad \int \sqrt[3]{x} \log x \, dx &= \int \left(\frac{3}{4} x^{\frac{4}{3}} \right)' \log x \, dx \\
 &= \frac{3}{4} x^{\frac{4}{3}} \log x - \int \frac{3}{4} x^{\frac{4}{3}} (\log x)' \, dx \\
 &= \frac{3}{4} x^{\frac{4}{3}} \log x - \int \frac{3}{4} x^{\frac{1}{3}} \, dx \\
 &= \frac{3}{4} x^{\frac{4}{3}} \log x - \frac{9}{16} x^{\frac{4}{3}} + C \\
 &= \frac{3}{4} x \sqrt[3]{x} \left(\log x - \frac{3}{4} \right) + C
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad \int x 2^x \, dx &= \int \left(\frac{2^x}{\log 2} \right)' \cdot x \, dx \\
 &= \frac{x \cdot 2^x}{\log 2} - \int \frac{2^x}{\log 2} (x)' \, dx \\
 &= \frac{x \cdot 2^x}{\log 2} - \frac{1}{\log 2} \int 2^x \, dx \\
 &= \frac{x \cdot 2^x}{\log 2} - \frac{2^x}{(\log 2)^2} + C
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad \int_0^\pi \sqrt{1 - \cos x} \, dx &= \int_0^\pi \sqrt{1 - \left(1 - 2 \sin^2 \frac{x}{2} \right)} \, dx \\
 &= \int_0^\pi \sqrt{2} \sin \frac{x}{2} \, dx \\
 &= \sqrt{2} \left[-2 \cos \frac{x}{2} \right]_0^\pi = 2\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad \int_0^1 \frac{1}{e^x + 1} \, dx \quad e^x + 1 = t \quad \text{とおくと} \\
 e^x = t - 1 \quad \begin{array}{|c|c|} \hline x & 0 \rightarrow 1 \\ \hline t & 0 \rightarrow e + 1 \\ \hline \end{array} \\
 e^x dx = dt \\
 \therefore dx = \frac{dt}{e^x} = \frac{dt}{t-1}
 \end{aligned}$$

$$\begin{aligned}
 (\text{与式}) &= \int_2^{e+1} \frac{dt}{t(t-1)} = \int_2^{e+1} \left(\frac{1}{t-1} - \frac{1}{t} \right) dt \\
 &= \left[\log |t-1| - \log |t| \right]_2^{e+1} \\
 &= \log e - \log(e+1) + \log 2 = \log \frac{2e}{e+1}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad \sin \theta &= 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2} = \frac{2 \sin \frac{\theta}{2} \cos^2 \frac{\theta}{2}}{\cos \frac{\theta}{2}} \\
 &= 2 \tan \frac{\theta}{2} \cdot \frac{1}{1 + \tan^2 \frac{\theta}{2}} = \frac{2x}{1+x^2}
 \end{aligned}$$

$$\begin{aligned}
 \cos \theta &= 2 \cos^2 \frac{\theta}{2} - 1 = \frac{2}{1 + \tan^2 \frac{\theta}{2}} - 1 \\
 &= \frac{2}{1+x^2} - 1 = \frac{1-x^2}{1+x^2}
 \end{aligned}$$

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{1 + \sin \theta + \cos \theta}$$

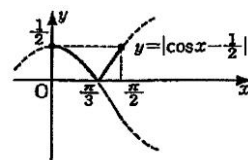
$$\tan \frac{\theta}{2} = x \quad \text{とおくと} \quad d\theta = 2 \cos^2 \frac{\theta}{2} \, dx$$

$$\begin{array}{|c|c|} \hline \theta & 0 \rightarrow \frac{\pi}{2} \\ \hline x & 0 \rightarrow 1 \\ \hline \end{array} \quad = \frac{2}{1+x^2} \, dx$$

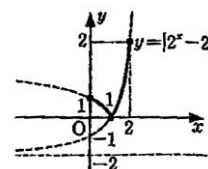
$$\begin{aligned}
 (\text{与式}) &= \int_0^1 \frac{1}{1 + \frac{2x}{1+x^2} + \frac{1-x^2}{1+x^2}} \cdot \frac{2}{1+x^2} \, dx \\
 &= \int_0^1 \frac{1}{1+x} \, dx = \left[\log |1+x| \right]_0^1 = \log 2
 \end{aligned}$$

3

$$\begin{aligned}
 (1) \quad \int_0^{\frac{\pi}{2}} \left| \cos x - \frac{1}{2} \right| dx &= \int_0^{\frac{\pi}{3}} \left(\cos x - \frac{1}{2} \right) dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\frac{1}{2} - \cos x \right) dx \\
 &= \left[\sin x - \frac{1}{2} x \right]_0^{\frac{\pi}{3}} + \left[\frac{1}{2} x - \sin x \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \\
 &= \left(\frac{\sqrt{3}}{2} - \frac{\pi}{6} \right) + \left(\frac{\pi}{4} - 1 \right) - \left(\frac{\pi}{6} - \frac{\sqrt{3}}{2} \right) = \sqrt{3} - 1 - \frac{\pi}{12}
 \end{aligned}$$



$$\begin{aligned}
 (2) \quad \int_0^2 |2^x - 2| dx &= \int_0^1 (2 - 2^x) dx + \int_1^2 (2^x - 2) dx \\
 &= \left[2x - \frac{2^x}{\log 2} \right]_0^1 + \left[\frac{2^x}{\log 2} - 2x \right]_1^2 \\
 &= \left(2 - \frac{2}{\log 2} \right) + \frac{1}{\log 2} + \left(\frac{4}{\log 2} - 4 \right) - \left(\frac{2}{\log 2} - 2 \right) = \frac{1}{\log 2}
 \end{aligned}$$



4

$$\begin{aligned}
 (1) \quad \int_0^1 x \cos \pi x \, dx &= \int_0^1 x \left(\frac{1}{\pi} \sin \pi x \right)' dx \\
 &= \left[x \frac{1}{\pi} \sin \pi x \right]_0^1 - \frac{1}{\pi} \int_0^1 \sin \pi x \, dx \\
 &= 0 + \frac{1}{\pi^2} \left[\cos \pi x \right]_0^1 = -\frac{2}{\pi^2}
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad I &= \int_0^1 \left\{ \cos^2 \pi x - (ax + b)^2 \right\} dx \\
 &= \int_0^1 \left\{ \cos^2 \pi x - 2(ax + b) \cos \pi x + (ax + b)^2 \right\} dx
 \end{aligned}$$

$$\begin{aligned}
 \text{ここで} \quad \int_0^1 \cos^2 \pi x \, dx &= \int_0^1 \frac{1 + \cos 2\pi x}{2} dx \\
 &= \frac{1}{2} \left[x + \frac{1}{2\pi} \sin 2\pi x \right]_0^1 = \frac{1}{2} \\
 \int_0^1 2(ax + b) \cos \pi x \, dx &= 2a \int_0^1 x \cos \pi x \, dx + 2b \int_0^1 \cos \pi x \, dx \\
 &= 2a \cdot \left(-\frac{2}{\pi^2} \right) + 2b \left[\frac{1}{\pi} \sin \pi x \right]_0^1 = -\frac{4a}{\pi^2}
 \end{aligned}$$

$$\begin{aligned}
 \int_0^1 (ax + b)^2 dx &= \int_0^1 (a^2 x^2 + 2abx + b^2) dx \\
 &= \left[\frac{a^2}{3} x^3 + abx^2 + b^2 x \right]_0^1 = \frac{a^2}{3} + ab + b^2
 \end{aligned}$$

$$\text{よって} \quad I = \frac{1}{2} + \frac{4a}{\pi^2} + \frac{a^2}{3} + ab + b^2$$

$$(3) \quad I = \left(b + \frac{a}{2}\right)^2 + \frac{a^2}{12} + \frac{4a}{\pi^2} + \frac{1}{2}$$

$$= \left(b + \frac{a}{2}\right)^2 + \frac{1}{12} \left(a + \frac{24}{\pi^2}\right)^2 + \frac{1}{2} - \frac{48}{\pi^4}$$

これより $b + \frac{a}{2} = 0$ かつ $a + \frac{24}{\pi^2} = 0$ のとき、最小値をとる。

したがって $a = -\frac{24}{\pi^2}$, $b = \frac{12}{\pi^2}$ のとき 最小値 $\frac{1}{2} - \frac{48}{\pi^4}$

5

$$(1) \quad f(x) = x^2 - \int_0^x (x-t)f'(t)dt \quad (2) \quad (e^x f(x))' = e^x f(x) + e^x f'(x)$$

$$= x^2 - x \int_0^x f'(t)dt + \int_0^x t f'(t)dt$$

$$= e^x f(x) + e^x (2x - f(x))$$

$$= e^x f(x) + 2xe^x - e^x f(x) = 2xe^x \quad (\text{証明終})$$

$$f(0) = 0^2 - 0 + \int_0^0 t f'(t)dt = 0$$

$$f'(x) = 2x - \int_0^x f'(t)dt - x f'(x) + x f'(x)$$

$$= 2x - \left[f(t)\right]_0^x = 2x - f(x) \quad (\text{証明終})$$

$$(3) \quad (e^x f(x))' = 2xe^x \quad \text{の両辺を } x \text{ で積分すると}$$

$$\int (e^x f(x))' dx = \int 2xe^x dx$$

$$e^x f(x) = 2 \int (e^x)' x dx$$

$$= 2e^x x - 2 \int e^x \cdot 1 dx = 2xe^x - 2e^x + C$$

$$\therefore f(x) = 2x - 2 + Ce^{-x}$$

$$f(0) = 0 \quad \text{より} \quad f(0) = -2 + Ce^0 = 0$$

$$\therefore C = 2$$

$$\text{よって} \quad f(x) = 2x - 2 + 2e^{-x}$$

6

$$(1) \quad \begin{array}{c} y \\ \uparrow \\ 1 \\ \sqrt{3}/2 \\ \pi/6 \quad \pi/4 \quad \pi/2 \\ x \end{array} \quad \begin{array}{l} y = \sin 2x \\ y = \cos x \end{array}$$

$$(2) \quad \int_0^{\pi/2} \sin 2x dx = \left[-\frac{1}{2} \cos 2x\right]_0^{\pi/2}$$

$$= -\frac{1}{2}(-1 - 1) = 1$$

$$(3) \quad \sin 2x = \cos x \quad \text{より} \quad 2 \sin x \cos x = \cos x$$

$$\cos x (2 \sin x - 1) = 0 \quad \therefore \sin x = \frac{1}{2}, \quad \cos x = 0$$

$$0 \leq x \leq \frac{\pi}{2} \quad \text{だから} \quad x = \frac{\pi}{6}, \quad \frac{\pi}{2}$$

$$\text{よって、交点は} \quad \left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right), \quad \left(\frac{\pi}{2}, 0\right)$$

$$(4) \quad \int_{\pi/6}^{\pi/2} (\sin 2x - \cos x) dx = \left[-\frac{1}{2} \cos 2x - \sin x\right]_{\pi/6}^{\pi/2}$$

$$= \left(\frac{1}{2} - 1\right) - \left(-\frac{1}{4} - \frac{1}{2}\right) = \frac{1}{4}$$

$$\begin{aligned}
 (5) \quad \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^2 2x \, dx - \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2 x \, dx &= \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{1 - \cos 4x}{2} - \frac{1 + \cos 2x}{2} \right) dx \\
 &= \pi \left[-\frac{1}{8} \sin 4x - \frac{1}{4} \sin 2x \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \\
 &= \pi \left(\frac{\sqrt{3}}{16} + \frac{\sqrt{3}}{8} \right) = \frac{3\sqrt{3}}{16} \pi
 \end{aligned}$$

7 2つの曲線は右図のようになる。この2曲線の交点を $x = \alpha$ とすると $a \cos \alpha = \sin \alpha$

$$0 < \alpha < \frac{\pi}{2} \quad \text{より} \quad \sin \alpha > 0, \quad \cos \alpha > 0$$

よって $a \cos \alpha = \sin \alpha$ と

$\sin^2 \alpha + \cos^2 \alpha = 1$ を連立して解いて

$$\sin \alpha = \frac{a}{\sqrt{1+a^2}}, \quad \cos \alpha = \frac{1}{\sqrt{1+a^2}} \quad \dots \textcircled{1}$$

$S_1 = S_2$ となればよいから

$$\int_0^\alpha (a \cos x - \sin x) \, dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} a \cos x \, dx$$

$$\left[a \sin x + \cos x \right]_0^\alpha = \frac{1}{2} \left[a \sin x \right]_0^{\frac{\pi}{2}}$$

$$a \sin \alpha + \cos \alpha - 1 = \frac{a}{2}$$

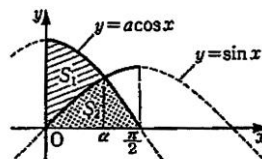
$$2a \sin \alpha + 2 \cos \alpha = a + 2$$

$$\textcircled{1} \text{を代入して} \quad \frac{2a^2}{\sqrt{1+a^2}} + \frac{2}{\sqrt{1+a^2}} = a + 2$$

$$2\sqrt{a^2+1} = a + 2$$

両辺2乗して $3a^2 - 4a = 0$

$$a(3a - 4) = 0 \quad a > 0 \quad \text{だから} \quad a = \frac{4}{3}$$



8 右図のように、球とワイングラスの接点 T の座標は $T(2, 4)$ である。

球の中心を $C(0, a)$ 、 T における接線を l とすると $CT \perp l$ である。

$$y' = 2x \text{ より } l \text{ の傾きは } 4 \quad CT \text{ の傾きは } \frac{4-a}{2-0}$$

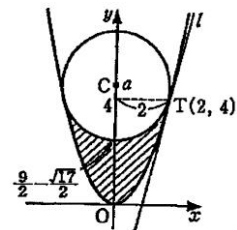
$$\therefore 4 \cdot \frac{4-a}{2} = -1 \text{ より } a = \frac{9}{2}$$

$$\text{球の半径は } CT = \sqrt{2^2 + \left(4 - \frac{9}{2}\right)^2} = \sqrt{\frac{17}{4}}$$

図の円の方程式（球を中心で切った断面）は

$$x^2 + \left(y - \frac{9}{2}\right)^2 = \frac{17}{4} \quad \text{求める体積は}$$

$$\begin{aligned} V &= \pi \int_0^4 y \, dy - \pi \int_{\frac{9}{2} - \frac{\sqrt{17}}{2}}^{\frac{9}{2}} \left\{ \frac{17}{4} - \left(y - \frac{9}{2}\right)^2 \right\} dy \\ &= \pi \left[\frac{1}{2} y^2 \right]_0^4 - \pi \left[\frac{17}{4} y - \frac{1}{3} \left(y - \frac{9}{2}\right)^3 \right]_{\frac{9}{2} - \frac{\sqrt{17}}{2}}^{\frac{9}{2}} \\ &= 8\pi - \pi \left\{ \frac{17}{4} \left(4 - \frac{9}{2} + \frac{\sqrt{17}}{2}\right) - \frac{1}{3} \left(-\frac{1}{8} + \frac{17\sqrt{17}}{8}\right) \right\} \\ &= \pi \left(8 + \frac{17}{8} - \frac{17\sqrt{17}}{8} - \frac{1}{24} + \frac{17\sqrt{17}}{24} \right) = \frac{\pi}{12} (121 - 17\sqrt{17}) \end{aligned}$$



9

- (1) 楕円を x 軸のまわりに回転してできる
立体の体積を V_1 、円柱（半径 b 、高さ $2a$ ）
の体積を V_2 とすると

$$\begin{aligned} V_1 &= \pi \int_{-2}^2 y^2 \, dx = 2\pi \int_0^2 \left(1 - \frac{x^2}{4}\right) dx \\ &= 2\pi \left[x - \frac{x^3}{12} \right]_0^2 = 2\pi \left(2 - \frac{2}{3}\right) = \frac{8}{3}\pi \end{aligned}$$

$$V_2 = \pi b^2 \cdot 2a = 2ab^2\pi$$

ここで、点 (a, b) は $\frac{x^2}{4} + y^2 = 1$ 上の点だから

$$\frac{a^2}{4} + b^2 = 1 \quad \therefore b^2 = 1 - \frac{a^2}{4} \quad \text{より}$$

$$V_2 = 2a \left(1 - \frac{a^2}{4}\right) \pi = 2 \left(a - \frac{a^3}{4}\right) \pi$$

$$\text{よって } V = V_1 - V_2 = \frac{8}{3}\pi - 2 \left(a - \frac{a^3}{4}\right) \pi$$

$$V = \left(\frac{8}{3} - 2a + \frac{a^3}{2}\right) \pi \quad (0 < a < 2)$$

$$\begin{aligned} (2) \quad V' &= \left(-2 + \frac{3}{2}a^2\right)\pi \\ &= \frac{3}{2} \left(a - \frac{2\sqrt{3}}{3}\right) \left(a + \frac{2\sqrt{3}}{3}\right) \pi \end{aligned}$$

a	0	...	$\frac{2\sqrt{3}}{3}$...	2
V'		-	0	+	
V		↘	極小	↗	

$$a = \frac{2\sqrt{3}}{3} \text{ のとき}$$

$$\begin{aligned} V &= \left(\frac{8}{3} - \frac{4\sqrt{3}}{3} + \frac{4\sqrt{3}}{9}\right) \pi \\ &= \frac{8(3 - \sqrt{3})}{9} \pi \end{aligned}$$

増減表より最小値は $a = \frac{2\sqrt{3}}{3}$ のとき

$$\frac{8(3 - \sqrt{3})}{9} \pi$$

(1) 2 曲線の交点の x 座標は

$$e^x = ne^{-x} \text{ より}$$

$$e^{2x} = n \Leftrightarrow 2x = \log n$$

$$\therefore x = \frac{1}{2} \log n$$

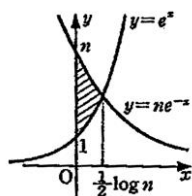
$$S_n = \int_0^{\frac{1}{2} \log n} (ne^{-x} - e^x) dx$$

$$= \left[-ne^{-x} - e^x \right]_0^{\frac{1}{2} \log n}$$

$$= \left(-ne^{-\frac{1}{2} \log n} - e^{\frac{1}{2} \log n} \right) - (-n - 1)$$

$$= -n \cdot n^{-\frac{1}{2}} - n^{\frac{1}{2}} + n + 1 = n - 2\sqrt{n} + 1$$

$$\therefore S_n = (\sqrt{n} - 1)^2$$



$$(2) \lim_{n \rightarrow \infty} (S_{n+1} - S_n)$$

$$= \lim_{n \rightarrow \infty} \left\{ (\sqrt{n+1} - 1)^2 - (\sqrt{n} - 1)^2 \right\}$$

$$= \lim_{n \rightarrow \infty} (\sqrt{n+1} + \sqrt{n} - 2)(\sqrt{n+1} - \sqrt{n})$$

$$\frac{(\sqrt{n+1} + \sqrt{n} - 2)(\sqrt{n+1} - \sqrt{n})(\sqrt{n+1} + \sqrt{n})}{\sqrt{n+1} + \sqrt{n}}$$

と変形すると

$$= \lim_{n \rightarrow \infty} \left(1 - \frac{2}{\sqrt{n+1} + \sqrt{n}} \right) = 1$$