

### 3章 集合と論理と統計

#### 13 講 確率

##### 練習問題

##### ① 基本

- (1)  $A = \{2, 4, 6\}$  (2)  $\{3, 4, 5, 6\}$
- (3)  $P \subset \Omega = \{1, 2, 6\}$  から  $P \cap \{3, 4, 5, 6\} = \phi$  となる  
 $P$  は  $\phi, \{1\}, \{2\}, \{1, 2\}$
- (4)  $P(\{2, 3, 4, 5, 6\}) = \frac{5}{6}$  (5)  $P(\{4, 6\}) = \frac{1}{3}$
- (6)  $A \cap C = \{6\}$  より  $(A \cap C) \cup B = \{6\} \cup \{3, 4, 5, 6\} = \{3, 4, 5, 6\}$
- (7) 集合の分配法則より  $(A \cup C) \cap (C \cup B) = \{A \cap C\} \cup B = \{3, 4, 5, 6\}$   
 (もちろん直接求めてもよい)
- (8)  $P((A \cup B) \cap (C \cap B)) = P(\{3, 4, 5, 6\}) = \frac{2}{3}$

##### ② 基本

(1)

X	0	1	2	3	4
確率	$\left(\frac{1}{2}\right)^4$	$4\left(\frac{1}{2}\right)^4$	$6\left(\frac{1}{2}\right)^4$	$4\left(\frac{1}{2}\right)^4$	$1\left(\frac{1}{2}\right)^4$

- (2)  $E(X) = 0 \times \left(\frac{1}{2}\right)^4 + 1 \times 4 \times \left(\frac{1}{2}\right)^4 + \dots + 4 \times \left(\frac{1}{2}\right)^4 = 2$
- (3)  $E(X^2) = 0^2 \times \left(\frac{1}{2}\right)^4 + 1^2 \times 4 \times \left(\frac{1}{2}\right)^4 + \dots + 4^2 \times \left(\frac{1}{2}\right)^4 = 5$   
 $\therefore V(X) = E(X^2) - (E(X))^2 = 5 - 2^2 = 1$
- (4)  $V(4 - 5X) = 25V(X) = 25$

③ 標準

(1)

$X$	0	2
確率	$\frac{2}{3}$	$\frac{1}{3}$

$Y$	0	3
確率	$\frac{3}{4}$	$\frac{1}{4}$

$$(2) \quad E(X) = \frac{2}{3}, \quad E(X^2) = \frac{4}{3} \quad \therefore V(X) = E(X^2) - (E(X))^2 = \frac{4}{3} - \left(\frac{2}{3}\right)^2 = \frac{8}{9}$$

$$(3) \quad E(Y) = \frac{3}{4}, \quad E(Y^2) = \frac{9}{4} \quad \therefore V(Y) = \frac{9}{4} - \left(\frac{3}{4}\right)^2 = \frac{27}{16}$$

$$(4) \quad V(3-4X) = (-4)^2 V(X) = 16 \times \frac{8}{9} = \frac{128}{9}$$

(5)  $X+Y$  のとる値  $= \{0, 2, 3, 5\}$  より

$X+Y$	0	2	3	5
確率	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{12}$

$$(6) \quad E(XY) = 2 \times 3 \times \frac{1}{12} = \frac{1}{2}$$

$$\therefore \text{Cov}(X, Y) = E(XY) - E(X)E(Y) = \frac{1}{2} - \frac{2}{3} \times \frac{3}{4} = 0$$

$$(7) \quad E(X+Y) = 0 \times \frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{6} + 5 \times \frac{1}{12} = \frac{17}{12}$$

$$\left( \text{別解} \quad E(X) + E(Y) = \frac{2}{3} + \frac{3}{4} = \frac{17}{12} \text{ でももちろんよい} \right)$$

分散時 (5)を用いてもよいがやや面倒なので

$$V(X+Y) = V(X) + 2\text{Cov}(X, Y) + V(Y) = \frac{8}{9} + 2 \times 0 + \frac{27}{16} = \frac{371}{144}$$

④ 標準

$$x+y = 1 - \frac{1}{4} - \frac{1}{12} = \frac{2}{3} \quad \therefore P(X=0 \cap Y=0) = P(X=0)P(Y=0) = \frac{2}{3} \times \left(x + \frac{1}{4}\right)$$

$$\therefore x = \frac{1}{2} \quad \therefore y = \frac{1}{6}$$

また  $x = \frac{1}{2}$   $y = \frac{1}{6}$  なら, 他の  $i, j$  に対しても  $P(X=i \cap Y=j) = P(X=i)P(Y=j)$  となる。