新版微分積分 I 演習 解答

1 節　関数の極限

55

(1) $\displaystyle\lim_{x\to 2}\left(2x^2-3x-4\right)=2\cdot 2^2-3\cdot 2-4=-2$

(2) $\displaystyle\lim_{x\to -1}\left(x^3-2x+2\right)=(-1)^3-2\cdot(-1)+2=3$

(3) $\displaystyle\lim_{x\to 1}\sqrt{3x-1}=\sqrt{3\cdot 1-1}=\sqrt{2}$

(4) $\displaystyle\lim_{x\to -2}\sqrt{2-x}=\sqrt{2-(-2)}=2$

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(1) $\displaystyle\lim_{x\to 0}\frac{x^2+2x}{2x}=\lim_{x\to 0}\frac{\cancel{x}(x+2)}{2\cancel{x}}=\lim_{x\to 0}\frac{x+2}{2}=1$

(2) $\displaystyle\lim_{x\to 3}\frac{x^2-9}{x-3}=\lim_{x\to 3}\frac{(x+3)\cancel{(x-3)}}{\cancel{x-3}}=\lim_{x\to 3}(x+3)=6$

(3) $\displaystyle\lim_{x\to -2}\frac{x^2-x-6}{x+2}=\lim_{x\to -2}\frac{\cancel{(x+2)}(x-3)}{\cancel{x+2}}=\lim_{x\to -2}(x-3)=-5$

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(1) $\displaystyle\lim_{x\to 1}\frac{x^2+x-2}{x^2-x}=\lim_{x\to 1}\frac{(x+2)\cancel{(x-1)}}{x\cancel{(x-1)}}=\lim_{x\to 1}\frac{x+2}{x}=3$

(2) $\displaystyle\lim_{x\to -1}\frac{x^3+1}{x+1}=\lim_{x\to -1}\frac{\cancel{(x+1)}(x^2-x+1)}{\cancel{x+1}}=\lim_{x\to -1}(x^2-x+1)=3$

(3) $\displaystyle\lim_{x\to 2}\frac{x^3-3x^2+4}{x^2-4x+4}=\lim_{x\to 2}\frac{(x+1)\cancel{(x-2)^2}}{\cancel{(x-2)^2}}=\lim_{x\to 2}(x+1)=3$

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(1) $\displaystyle\lim_{x\to 0}\frac{1}{x}\left(1-\frac{1}{x+1}\right)=\lim_{x\to 0}\left(\frac{1}{\cancel{x}}\cdot\frac{\cancel{x}}{x+1}\right)=\lim_{x\to 0}\frac{1}{x+1}=1$

(2) $\displaystyle\lim_{x\to 0}\frac{1}{x}\left(2+\frac{4}{x-2}\right)=\lim_{x\to 0}\left(\frac{1}{\cancel{x}}\cdot\frac{2\cancel{x}}{x-2}\right)=-1$

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(1) $\displaystyle\lim_{x\to 2}\frac{\sqrt{x-1}-1}{x-2}=\lim_{x\to 2}\frac{\left(\sqrt{x-1}-1\right)\left(\sqrt{x-1}+1\right)}{(x-2)\left(\sqrt{x-1}+1\right)}=\lim_{x\to 2}\frac{\cancel{x-2}}{\cancel{(x-2)}\left(\sqrt{x-1}+1\right)}=\lim_{x\to 2}\frac{1}{\sqrt{x-1}+1}$

$\displaystyle=\frac{1}{2}$

(2) $\displaystyle\lim_{x\to -2}\frac{\sqrt{x+11}-3}{x+2}=\lim_{x\to -2}\frac{\left(\sqrt{x+11}-3\right)\left(\sqrt{x+11}+3\right)}{(x+2)\left(\sqrt{x+11}+3\right)}=\lim_{x\to -2}\frac{\cancel{x+2}}{\cancel{(x+2)}\left(\sqrt{x+11}+3\right)}$

$\displaystyle=\lim_{x\to -2}\frac{1}{\sqrt{x+11}+3}=\frac{1}{6}$

(3) $\displaystyle\lim_{x\to 1}\frac{2\sqrt{x}-\sqrt{3x+1}}{x-1}=\lim_{x\to 1}\frac{\left(2\sqrt{x}-\sqrt{3x+1}\right)\left(2\sqrt{x}+\sqrt{3x+1}\right)}{(x-1)\left(2\sqrt{x}+\sqrt{3x+1}\right)}=\lim_{x\to 1}\frac{\cancel{x-1}}{\cancel{(x-1)}\left(2\sqrt{x}+\sqrt{3x+1}\right)}$

$\displaystyle=\lim_{x\to 1}\frac{1}{2\sqrt{x}+\sqrt{3x+1}}=\frac{1}{4}$

(4) $\displaystyle\lim_{x\to 3}\frac{\sqrt{3x}-\sqrt{2x+3}}{x-3}=\lim_{x\to 3}\frac{\left(\sqrt{3x}-\sqrt{2x+3}\right)\left(\sqrt{3x}+\sqrt{2x+3}\right)}{(x-3)\left(\sqrt{3x}+\sqrt{2x+3}\right)}=\lim_{x\to 3}\frac{\cancel{x-3}}{\cancel{(x-3)}\left(\sqrt{3x}+\sqrt{2x+3}\right)}$

$\displaystyle=\lim_{x\to 3}\frac{1}{\sqrt{3x}+\sqrt{2x+3}}=\frac{1}{6}$

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(1) 与式が成り立つには $\displaystyle\lim_{x\to 1}(x^2+ax+b)=0$　$\therefore$　$1+a+b=0$　$\therefore$　$b=-a-1$　… ①

$\displaystyle\therefore\ \lim_{x\to 1}\frac{x^2+ax+b}{x-1}=\lim_{x\to 1}\frac{x^2+ax+(-a-1)}{x-1}=\lim_{x\to 1}\frac{\cancel{(x-1)}(x+a+1)}{\cancel{x-1}}=\lim_{x\to 1}(x+a+1)=4$

$\therefore$　$a+2=4$　$\therefore$　$a=2$　これを ① に代入して　$b=-3$

以上より　$a=2,\ b=-3$

(2) 与式が成り立つには $\lim_{x\to 2}(x^2+ax+b)=0$ $\quad\therefore\quad 4+2a+b=0$ $\quad\therefore\quad b=-2a-4 \quad\cdots ①$

$$\therefore\quad \lim_{x\to 2}\frac{x^2+ax+b}{x-2}=\lim_{x\to 2}\frac{x^2+ax+(-2a-4)}{x-2}=\lim_{x\to 2}\frac{\cancel{(x-2)}(x+a+2)}{\cancel{x-2}}=\lim_{x\to 2}(x+a+2)=-1$$

$\therefore\quad a+4=-1 \quad\therefore\quad a=-5$ これを ① に代入して $b=6$

以上より $a=-5,\ b=6$

(3) 与式が成り立つには $\lim_{x\to -2}\left(a\sqrt{x+3}+b\right)=0$ $\quad\therefore\quad a+b=0$ $\quad\therefore\quad b=-a \quad\cdots ①$

$$\therefore\quad \lim_{x\to -2}\frac{a\sqrt{x+3}+b}{x+2}=\lim_{x\to -2}\frac{a\sqrt{x+3}-a}{x+2}=\lim_{x\to -2}\frac{a\cancel{(x+2)}}{\cancel{(x+2)}\left(\sqrt{x+3}+1\right)}$$

$=\lim_{x\to -2}\dfrac{a}{\sqrt{x+3}+1}=1$ $\quad\therefore\quad \dfrac{a}{2}=1$ $\quad\therefore\quad a=2$ これを ① に代入して $b=-2$

以上より $a=2,\ b=-2$

(4) 与式が成り立つには $\lim_{x\to 1}\left(a\sqrt{x+3}+b\right)=0$ $\quad\therefore\quad 2a+b=0$ $\quad\therefore\quad b=-2a \quad\cdots ①$

$$\therefore\quad \lim_{x\to 1}\frac{x^2+ax+b}{x-1}=\lim_{x\to 1}\frac{a\sqrt{x+3}-2a}{x-1}=\lim_{x\to 1}\frac{a\cancel{(x-1)}}{\cancel{(x-1)}\left(\sqrt{x+3}+2\right)}=\lim_{x\to 1}\frac{a}{\left(\sqrt{x+3}+2\right)}=1$$

$\therefore\quad \dfrac{a}{4}=1$ $\quad\therefore\quad a=4$ これを ① に代入して $b=-8$

以上より $a=4,\ b=-8$

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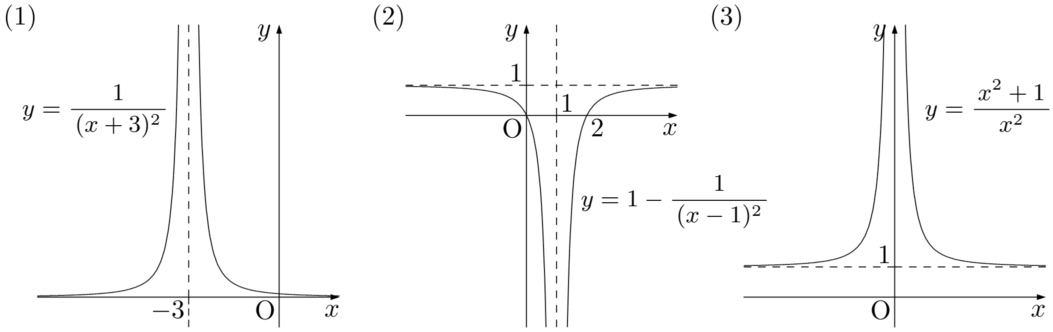
(1) グラフより ∞

(2) グラフより $-\infty$

(3) グラフより ∞

グラフは下図の通り



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(1) $\displaystyle\lim_{x\to 1-0}\frac{|x-1|}{x-1}=\lim_{x\to 1-0}\frac{-\cancel{(x-1)}}{\cancel{x-1}}=\lim_{x\to 1-0}(-1)=-1$

(2) $\displaystyle\lim_{x\to 1+0}\frac{|1-x|}{x-1}=\lim_{x\to 1+0}\frac{\cancel{x-1}}{\cancel{x-1}}=\lim_{x\to 1+0}1=1$

(3) $\displaystyle\lim_{x\to -0}\frac{\sqrt{x^2}}{x}=\lim_{x\to -0}\frac{|x|}{x}=\lim_{x\to -0}\frac{-\cancel{x}}{\cancel{x}}=\lim_{x\to -0}(-1)=-1$

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(1) $\displaystyle\lim_{x\to\infty}\frac{1}{x-1}=0$

(2) $\displaystyle\lim_{x\to -\infty}\frac{1}{x^2-1}=0$

(3) $\displaystyle\lim_{x\to\infty}\left(1-\frac{1}{x^3}\right)=1$

(4) $\displaystyle\lim_{x\to\infty}(x^3-x)=\lim_{x\to\infty}x^3\left(1-\frac{1}{x^2}\right)=\infty$

(5) $\displaystyle\lim_{x\to -\infty}(x^3-x^2-x)=\lim_{x\to -\infty}x^3\left(1-\frac{1}{x}-\frac{1}{x^2}\right)=-\infty$

(6) $\displaystyle\lim_{x\to -\infty}\left(x^2-\frac{1}{x}\right)=\lim_{x\to -\infty}x^2\left(1-\frac{1}{x^3}\right)=\infty$

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(1) $\displaystyle\lim_{x\to\infty}\frac{1-x^2}{1+x^2}=\lim_{x\to\infty}\frac{\frac{1}{x^2}-1}{\frac{1}{x^2}+1}=-1$

(2) $\displaystyle\lim_{x\to-\infty}\frac{8x^3+1}{x^3+x+1}=\lim_{x\to-\infty}\frac{8+\frac{1}{x^3}}{1+\frac{1}{x^2}+\frac{1}{x^3}}=8$

(3) $\displaystyle\lim_{x\to\infty}\frac{x^3-1}{x^2+1}=\lim_{x\to\infty}\frac{(x^2+1)x-(x+1)}{x^2+1}=\lim_{x\to\infty}\left(x-\frac{x+1}{x^2+1}\right)=\lim_{x\to\infty}\left(x-\frac{1+\frac{1}{x}}{x+\frac{1}{x}}\right)=\infty$

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(1) $\displaystyle\lim_{x\to\infty}\left(\frac{1}{3}\right)^x=0$

(2) $\displaystyle\lim_{x\to\infty}\frac{2^x-3^x}{3^x}=\lim_{x\to\infty}\left\{\left(\frac{2}{3}\right)^x-1\right\}=-1$

(3) $\displaystyle\lim_{x\to-\infty}\frac{2^x+3^x}{4^x}=\lim_{x\to-\infty}\left\{\left(\frac{1}{2}\right)^x+\left(\frac{3}{4}\right)^x\right\}=\infty$

(4) $\displaystyle\lim_{x\to-\infty}\frac{5^x}{4^x+5^x}=\lim_{x\to-\infty}\frac{1}{\left(\frac{4}{5}\right)^x+1}=0$

(5) $\displaystyle\lim_{x\to\infty}\frac{3^x+4^x}{3^x-4^x}=\lim_{x\to\infty}\frac{\left(\frac{3}{4}\right)^x+1}{\left(\frac{3}{4}\right)^x-1}=-1$

(6) $\displaystyle\lim_{x\to\infty}\frac{2^{-x}}{1+2^{-x}}=0$

66

(1) $\displaystyle\lim_{x\to\infty}\log_{\frac{1}{2}}x=-\infty$

(2) $\displaystyle\lim_{x\to\infty}\log_{0.1}\frac{1}{x}=\lim_{x\to\infty}\log_{0.1}x^{-1}=-\lim_{x\to\infty}\log_{0.1}x=\infty$

(3) $\displaystyle\lim_{x\to+0}\log_3 x=-\infty$

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(1) $\displaystyle\lim_{x\to0}\frac{\sin3x}{2x}=\lim_{x\to0}\left(\frac{3}{2}\cdot\frac{\sin3x}{3x}\right)=\frac{3}{2}$

(2) $\displaystyle\lim_{x\to0}\frac{\tan3x}{x}=\lim_{x\to0}\frac{\sin3x}{x\cos3x}=\lim_{x\to0}\left(\frac{\sin3x}{3x}\cdot\frac{3}{\cos3x}\right)=3$

(3) $\displaystyle\lim_{x\to0}\frac{\sin4x}{\sin2x}=\lim_{x\to0}\frac{\frac{\sin4x}{4x}}{\frac{1}{2}\cdot\frac{\sin2x}{2x}}=2$

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(1) $\displaystyle\lim_{x\to0}f(x)=\lim_{x\to0}(x^2-1)=-1\neq1=f(0)$ $\quad\therefore$ 連続でない

(2) $\displaystyle\lim_{x\to0}f(x)=\lim_{x\to0}\frac{x^2+x}{x^2-x}=\lim_{x\to0}\frac{x+1}{x-1}=-1=f(0)$ $\quad\therefore$ 連続である

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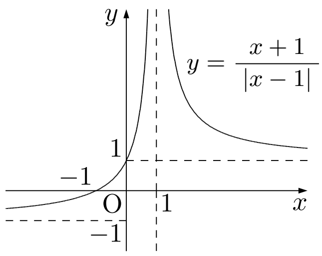
(1) $\displaystyle\lim_{x\to\infty}\frac{x^2+2x-3}{2-x}=\lim_{x\to\infty}\frac{(2-x)(-x-4)+5}{2-x}=\lim_{x\to\infty}\left(-x-4+\frac{5}{2-x}\right)=-\infty$

(2) $x>1$ のとき，$\displaystyle\frac{x+1}{|x-1|}=\frac{x+1}{x-1}=1+\frac{2}{x-1}$

$x<1$ のとき，$\displaystyle\frac{x+1}{|x-1|}=-\frac{x+1}{x-1}=-1-\frac{2}{x-1}$

図より $\displaystyle\lim_{x\to1}\frac{x+1}{|x-1|}=\infty$



(3) $t=\dfrac{1}{x}$ とおくと，$x \longrightarrow +0$ のとき $t \longrightarrow \infty$ だから

$$\lim_{x\to+0}\left(\frac{1}{2}\right)^{\frac{1}{x}}=\lim_{t\to\infty}\left(\frac{1}{2}\right)^{t}=0$$

(4) $\displaystyle\lim_{x\to\frac{\pi}{2}-0}\tan x=\infty$

(5) $\displaystyle\lim_{x\to\pi}\frac{\sin 2x}{\tan x}=\lim_{x\to\pi}\frac{2\cancel{\sin x}\cos x}{\dfrac{\cancel{\sin x}}{\cos x}}=\lim_{x\to\pi}2\cos^2 x=2$

(6) $\displaystyle\lim_{x\to\infty}\mathrm{Tan}^{-1}x=\frac{\pi}{2}$

(7) $\displaystyle\lim_{x\to+0}\log_2|x|=\lim_{x\to+0}\log_2 x=-\infty$

また，$t=-x$ とおくと，$x \longrightarrow -0$ のとき $t \longrightarrow +0$ だから

$$\lim_{x\to-0}\log_2|x|=\lim_{x\to-0}\log_2(-x)=\lim_{t\to+0}\log_2 t=-\infty$$

$\therefore\ \displaystyle\lim_{x\to0}\log_2|x|=-\infty$

(8) $\displaystyle\lim_{x\to0}\left\{\log_3(x^2+x)-\log_3 x\right\}=\lim_{x\to0}\log_3\frac{x^2+x}{x}=\lim_{x\to0}\log_3(x+1)=0$

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(1) $\displaystyle\lim_{x\to2}\frac{2x^2-3x-2}{3x^2-7x+2}=\lim_{x\to2}\frac{(2x+1)\cancel{(x-2)}}{(3x-1)\cancel{(x-2)}}=\lim_{x\to2}\frac{2x+1}{3x-1}=1$

(2) $\displaystyle\lim_{x\to0}\frac{\sin 4x\tan 3x}{2x^2}=\lim_{x\to0}\left(6\cdot\frac{\sin 4x}{4x}\cdot\frac{\sin 3x}{3x}\cdot\frac{1}{\cos 3x}\right)=6$

(3) $\displaystyle\lim_{x\to0}\frac{1-\cos 2x}{x\sin x}=\lim_{x\to0}\left(\frac{2}{x\sin x}\cdot\frac{1-\cos 2x}{2}\right)=\lim_{x\to0}\left(\frac{2}{x\sin x}\cdot\sin^2 x\right)=\lim_{x\to0}\left(2\cdot\frac{\sin x}{x}\right)=2$

(4) $t=\dfrac{1}{x}$ とおくと，$x \longrightarrow \infty$ のとき $t \longrightarrow +0$ だから

$$\lim_{x\to\infty}x\sin\frac{1}{x}=\lim_{t\to+0}\frac{\sin t}{t}=1$$

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(1) $\displaystyle\lim_{x\to4}\frac{x-4}{\sqrt{x}-2}=\lim_{x\to4}\frac{\cancel{(x-4)}\left(\sqrt{x}+2\right)}{\cancel{x-4}}=\lim_{x\to4}\left(\sqrt{x}+2\right)=4$

(2) $\displaystyle\lim_{x\to3}\frac{\sqrt{x+6}-3}{\sqrt{x+1}-2}=\lim_{x\to3}\frac{\cancel{(x-3)}\left(\sqrt{x+1}+2\right)}{\cancel{(x-3)}\left(\sqrt{x+6}+3\right)}=\lim_{x\to3}\frac{\sqrt{x+1}+2}{\sqrt{x+6}+3}=\frac{2}{3}$

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(1) $\displaystyle\lim_{x\to\infty}\left(x-\sqrt{x^2-2x}\right)=\lim_{x\to\infty}\frac{\cancel{x^2}-(\cancel{x^2}-2x)}{x+\sqrt{x^2-2x}}=\lim_{x\to\infty}\frac{2x}{x+\sqrt{x^2-2x}}=\lim_{x\to\infty}\frac{2}{1+\sqrt{1-\dfrac{2}{x}}}=1$

(2) $\displaystyle\lim_{x\to\infty}\left\{\log_2(2x+1)-\log_2 x\right\}=\lim_{x\to\infty}\log_2\frac{2x+1}{x}=\lim_{x\to\infty}\log_2\left(2+\frac{1}{x}\right)=1$

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(1) $\displaystyle\lim_{x\to0}f(x)=\lim_{x\to0}\frac{x^2+x}{x}=\lim_{x\to0}(x+1)=1\neq0=f(0)$　　$\therefore$ 連続でない

(2) $\displaystyle\lim_{x\to+0}f(x)=\lim_{x\to0}\frac{\sin x}{x}=1$　　$\displaystyle\lim_{x\to-0}f(x)=\lim_{x\to0}\frac{\sin x}{-x}=-1$

$\displaystyle\lim_{x\to+0}f(x)\neq\lim_{x\to-0}f(x)$ より、$\displaystyle\lim_{x\to0}f(x)$ は存在しない　　$\therefore$ 連続でない

(3) $t=-\dfrac{1}{x^2}$ とおくと，$x \longrightarrow 0$ のとき $t \longrightarrow -\infty$ だから

$\displaystyle\lim_{x\to0}f(x)=\lim_{x\to0}2^{-\frac{1}{x^2}}=\lim_{t\to-\infty}2^t=0=f(0)$　　$\therefore$ 連続である

(4) $t=\log_2|x|$ とおくと，$x \longrightarrow 0$ のとき $t \longrightarrow -\infty$ だから

$\displaystyle\lim_{x\to0}f(x)=\lim_{x\to0}2^{\log_2|x|}=\lim_{x\to-\infty}2^t=0=f(0)$　　$\therefore$ 連続である

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(1) 任意の $x\neq0$ に対し，$-1\leqq\cos\dfrac{1}{x}\leqq1$　　$\therefore$ $-|x|\leqq\cos\dfrac{1}{x}\leqq|x|$

また，$\displaystyle\lim_{x\to0}(-|x|)=\lim_{x\to0}|x|=0$ だから，はさみうちの原理より，$\displaystyle\lim_{x\to0}x\cos\frac{1}{x}=0$

(2) 任意の $x \neq 0$ に対し，$-1 \leqq \sin x \leqq 1$ $\quad \therefore \ -\dfrac{1}{|x|} \leqq \dfrac{\sin x}{x} \leqq \dfrac{1}{|x|}$

また，$\displaystyle\lim_{x\to\infty}\left(-\frac{1}{|x|}\right)=\lim_{x\to\infty}\left(\frac{1}{|x|}\right)=0$ だから，はさみうちの原理より，$\displaystyle\lim_{x\to\infty}\frac{\sin x}{x}=0$

(3) 任意の $x \neq 0$ に対し，$-1 \leqq \sin 2x \leqq 1$ $\quad \therefore \ -\dfrac{1}{|x|} \leqq \dfrac{\sin 2x}{x} \leqq \dfrac{1}{|x|}$

また，$\displaystyle\lim_{x\to-\infty}\left(-\frac{1}{|x|}\right)=\lim_{x\to-\infty}\left(\frac{1}{|x|}\right)=0$ だから，はさみうちの原理より，$\displaystyle\lim_{x\to-\infty}\frac{\sin 2x}{x}=0$

2節　導関数

75

(1) $\displaystyle f'(-1)=\lim_{h\to0}\frac{\{(-1+h)^2-(-1+h)\}-\{(-1)^2-(-1)\}}{h}=\lim_{h\to0}\frac{-3h+h^2}{h}=\lim_{h\to0}(-3+h)=-3$

(2) $\displaystyle f'(-1)=\lim_{h\to0}\frac{\{2(-1+h)^2-1\}-\{2(-1)^2-1\}}{h}=\lim_{h\to0}\frac{-4h+2h^2}{h}=\lim_{h\to0}(-4+2h)=-4$

(3) $\displaystyle f'(-1)=\lim_{h\to0}\frac{\{(-1+h)^3-1\}-\{(-1)^3-1\}}{h}=\lim_{h\to0}\frac{3h-3h^2+h^3}{h}=\lim_{h\to0}\left(3-3h+h^2\right)=3$

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(1) $\displaystyle f'(x)=\lim_{h\to0}\frac{\{2(x+h)+1\}-\{2x+1\}}{h}=\lim_{h\to0}\frac{2h}{h}=\lim_{h\to0}2=2$

(2) $\displaystyle f'(x)=\lim_{h\to0}\frac{\{(x+h)^2+(x+h)\}-\{x^2+x\}}{h}=\lim_{h\to0}\frac{2xh+h^2+h}{h}=\lim_{h\to0}(2x+h+1)=2x+1$

(3) $\displaystyle f'(x)=\lim_{h\to0}\frac{\{(x+h)^3-(x+h)\}-\{x^3-x\}}{h}=\lim_{h\to0}\frac{3x^2h+3xh^2+h^3-h}{h}$

$\displaystyle =\lim_{h\to0}\left(3x^2+3xh+h^2-1\right)=3x^2-1$

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(1) $y'=6x-5$

(2) $y'=-3x^2+4x+1$

(3) $y'=\left(9x^2-6x+1\right)'=18x-6$

(4) $y'=\left(x^3-3x^2+2x\right)'=3x^2-6x+2$

(5) $y'=\left(x^3-2x^2+x-2\right)'=3x^2-4x+1$

(6) $y'=\left(8x^3-12x^2+6x-1\right)'=24x^2-24x+6$

78

(1) $y'=3\cdot(x^2-2x-1)+(3x+2)(2x-2)=9x^2-8x-7$

(2) $y'=1\cdot(2x+1)(3x-1)+(x+1)\cdot2\cdot(3x-1)+(x+1)(2x+1)\cdot3=18x^2+14x$

79

(1) $y'=-\dfrac{4}{(4x-1)^2}$

(2) $y'=\dfrac{2(x+1)-(2x-1)\cdot1}{(x+1)^2}=\dfrac{3}{(x+1)^2}$

(3) $y'=\dfrac{1\cdot(x^2+2)-(x+1)\cdot2x}{(x^2+2)^2}=-\dfrac{x^2+2x-2}{(x^2+2)^2}$

80

(1) $y'=\left(x^{-4}\right)'=-4x^{-5}=-\dfrac{4}{x^5}$

(2) $y'=\left(2x^{-2}\right)'=-4x^{-3}=-\dfrac{4}{x^3}$

(3) $y'=\left(-\dfrac{1}{2}x^{-6}\right)'=3x^{-7}=\dfrac{3}{x^7}$

81

(1) $u=5x+4$ とおくと $y=u^2$ であり，$\dfrac{dy}{du}=2u$，$\dfrac{du}{dx}=5$ だから　$y'=2u\cdot5=10(5x+4)$

(2) $u=4x-1$ とおくと $y=u^3$ であり，$\dfrac{dy}{du}=3u^2$，$\dfrac{du}{dx}=4$ だから　$y'=3u^2\cdot4=12(4x-1)^2$

(3) $u=2x^2+1$ とおくと $y=u^4$ であり，$\dfrac{dy}{du}=4u^3$，$\dfrac{du}{dx}=4x$ だから　$y'=4u^3\cdot 4x=16x(2x^2+1)^3$

(4) $u=3x^2-x+1$ とおくと $y=u^3$ であり，$\dfrac{dy}{du}=3u^2$，$\dfrac{du}{dx}=6x-1$ だから

$y'=3u^2\cdot(6x-1)=3(6x-1)(3x^2-x+1)^2$

(5) $u=x-1$ とおくと $y=u^{-2}$ であり，$\dfrac{dy}{du}=-2u^{-3}$，$\dfrac{du}{dx}=1$ だから

$y'=-2u^{-3}\cdot 1=-\dfrac{2}{(x-1)^3}$

(6) $u=x^2+3$ とおくと $y=u^{-4}$ であり，$\dfrac{dy}{du}=-4u^{-5}$，$\dfrac{du}{dx}=2x$ だから

$y'=-4u^{-5}\cdot 2x=-\dfrac{8x}{(x^2+3)^5}$

82

(1) $y'=\left(x^{\frac{3}{2}}\right)'=\dfrac{3}{2}u^{\frac{1}{2}}=\dfrac{3}{2}\sqrt{x}$

(2) $y'=\left\{\left(x^2+1\right)^{\frac{1}{2}}\right\}'=\dfrac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}\cdot 2x=\dfrac{x}{\sqrt{x^2+1}}$

(3) $y'=\left\{\left(3x^2+1\right)^{\frac{1}{3}}\right\}'=\dfrac{1}{3}\left(3x^2+1\right)^{-\frac{2}{3}}\cdot 6x=\dfrac{2x}{\sqrt[3]{(3x^2+1)^2}}$

83

(1) 両辺を 2 乗して　$y^2=x+1$　　$\therefore$　$x=y^2-1$　　$\therefore$　$\dfrac{dx}{dy}=2y$　　$\therefore$　$y'=\dfrac{1}{2y}=\dfrac{1}{2\sqrt{x+1}}$

(2) 両辺を 3 乗して　$y^3=\dfrac{27}{x}$　　$\therefore$　$x=\dfrac{27}{y^3}$　　$\therefore$　$\dfrac{dx}{dy}=-\dfrac{81}{y^4}$

$y'=\dfrac{1}{-\dfrac{81}{y^4}}=-\dfrac{1}{81}y^4=-\dfrac{1}{81}\cdot\dfrac{81}{\sqrt[3]{x^4}}=-\dfrac{1}{\sqrt[3]{x^4}}$

84

(1) $y'=-\sin 2x\cdot 2=-2\sin 2x$

(2) $y'=\cos(1-x)\cdot(-1)=-\cos(1-x)$

(3) $y'=\dfrac{1}{\cos^2 3x}\cdot 3=\dfrac{3}{\cos^2 3x}$

(4) $y'=2\sin x\cdot\cos x=2\sin x\cos x$

(5) $y'=3\cos^2 x\cdot(-\sin x)=-3\cos^2 x\sin x$

(6) $y'=2\tan x\cdot\dfrac{1}{\cos^2 x}=\dfrac{2\tan x}{\cos^2 x}$

(7) $y'=-\dfrac{\cos x}{\sin^2 x}$

(8) $y'=-\dfrac{-\sin x}{\cos^2 x}=\dfrac{\sin x}{\cos^2 x}$

(9) $y'=\dfrac{-\sin x\cdot x-\cos x\cdot 1}{x^2}=-\dfrac{x\sin x+\cos x}{x^2}$

85

(1) $y'=\dfrac{1}{\sqrt{1-(2x)^2}}\cdot 2=\dfrac{2}{\sqrt{1-4x^2}}$

(2) $y'=-\dfrac{1}{\sqrt{1-(3x)^2}}\cdot 3=-\dfrac{3}{\sqrt{1-9x^2}}$

(3) $y'=\dfrac{1}{1+(2x)^2}\cdot 2=\dfrac{2}{1+4x^2}$

(4) $y'=\dfrac{1}{\sqrt{1-\left(\dfrac{x}{3}\right)^2}}\cdot\dfrac{1}{3}=\dfrac{1}{\sqrt{9-x^2}}$

(5) $y'=\dfrac{1}{1+\left(\dfrac{x}{2}\right)^2}\cdot\dfrac{1}{2}=\dfrac{2}{4+x^2}$

(6) $y'=\dfrac{1}{1+x}\cdot\dfrac{1}{2\sqrt{x}}=\dfrac{1}{2(x+1)\sqrt{x}}$

86

(1) $y' = \dfrac{1}{2x} \cdot 2 = \dfrac{1}{x}$

(2) $y' = \dfrac{1}{3x+1} \cdot 3 = \dfrac{3}{3x+1}$

(3) $y' = \dfrac{1}{x \log 3}$

(4) $y' = 3x^2 \log x + x^3 \cdot \dfrac{1}{x} = x^2(3\log x + 1)$

(5) $y' = 1 \cdot \log_2 x + x \cdot \dfrac{1}{x \log 2} = \dfrac{\log x}{\log 2} + \dfrac{1}{\log 2} = \dfrac{1}{\log 2}(\log x + 1)$

(6) $y' = \dfrac{\dfrac{1}{x} \cdot x - \log x \cdot 1}{x^2} = \dfrac{1 - \log x}{x^2}$

87

(1) $y' = \dfrac{1}{2x-1} \cdot 2 = \dfrac{2}{2x-1}$

(2) $y' = \dfrac{1}{x^2 - x} \cdot (2x-1) = \dfrac{2x-1}{x^2-x}$

(3) $y' = \dfrac{1}{\cos x} \cdot (-\sin x) = -\tan x$

88

両辺の対数をとると　$\log y = \log \dfrac{(x-2)^3}{(x-1)^2} = \log(x-2)^3 - \log(x-1)^2 = 3\log(x-2) - 2\log(x-1)$

この両辺を x で微分すると　$\dfrac{y'}{y} = \dfrac{3}{x-2} - \dfrac{2}{x-1} = \dfrac{x+1}{(x-2)(x-1)}$

$\therefore \quad y' = \dfrac{x+1}{(x-2)(x-1)} \cdot \dfrac{(x-2)^3}{(x-1)^2} = \dfrac{(x+1)(x-2)^2}{(x-1)^3}$

89

(1) $y' = e^{3x+1} \cdot 3 = 3e^{3x+1}$

(2) $y' = 1 \cdot e^x + xe^x = (x+1)e^x$

(3) $y' = 2^{1-x} \log 2 \cdot (-1) = -2^{1-x} \log 2$

90

(1) $y' = e^x \cos x + e^x(-\sin x) = e^x(\cos x - \sin x)$

(2) $y' = \dfrac{e^x(x+1) - e^x \cdot 1}{(x+1)^2} = \dfrac{xe^x}{(x+1)^2}$

(3) $y' = e^{-x^2} \cdot (-2x) = -2xe^{-x^2}$

91

(1) $y'' = \left(6x^2 - 6x + 4\right)' = 12x - 6$

(2) $y'' = \left(\dfrac{1}{1+x^2}\right)' = -\dfrac{2x}{(1+x^2)^2}$

(3) $y'' = (\sin x + x\cos x)' = \cos x + \cos x + x(-\sin x) = 2\cos x - x\sin x$

92

(1) $y''' = (5x^4 + 8x^3 - 9x^2)'' = (20x^3 + 24x^2 - 18x)' = 60x^2 + 48x - 18$

(2) $y''' = (2\cos 2x)'' = (-4\sin 2x)' = -8\cos 2x$

(3) $y''' = \left(\dfrac{3}{2}\sqrt{x}\right)'' = \left(\dfrac{3}{4\sqrt{x}}\right)' = -\dfrac{3}{8\sqrt{x^3}}$

93

(1) $y' = -e^{-x}, \quad y'' = e^{-x}, \quad y''' = -e^{-x}, \quad y^{(4)} = e^{-x}, \quad \cdots \therefore \quad y^{(n)} = (-1)^n e^{-x}$

(2) $y' = e^{2x} + x \cdot 2e^{2x} = (2x+1)e^{2x}, \quad y'' = 2e^{2x} + (2x+1) \cdot 2e^{2x} = 2(2x+2)e^{2x},$

$y''' = 2^2 e^{2x} + 2(2x+2) \cdot 2e^{2x} = 2^2(2x+3)e^{2x}, \quad \cdots \quad y^{(4)} = \quad \therefore \quad y^{(n)} = 2^{n-1}(2x+n)e^{2x}$

(3) $y' = -\dfrac{1}{(x-1)^2}, \quad y'' = \dfrac{2 \cdot 1}{(x-1)^3}, \quad y''' = -\dfrac{3 \cdot 2 \cdot 1}{(x-1)^4}, \quad y^{(4)} = \dfrac{4 \cdot 3 \cdot 2 \cdot 1}{(x-1)^5} \quad \cdots$

$\therefore \quad y^{(n)} = \dfrac{(-1)^n n!}{(x-1)^{n+1}}$

94

$$y' = e^{-x} + x\cdot\left(-e^{-x}\right) = (1-x)e^{-x}, \qquad y'' = -1\cdot e^{-x} + (1-x)\cdot\left(-e^{-x}\right) = (x-2)e^{-x}$$

$$\therefore\quad y'' + 2y' + y = (x-2)e^{-x} + 2\cdot(1-x)e^{-x} + xe^{-x} = 0$$

95

(1) $$y' = \lim_{h\to 0}\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h} = \lim_{h\to 0}\frac{\left(\sqrt[3]{x+h}-\sqrt[3]{x}\right)\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{x+h}\,\sqrt[3]{x}+\sqrt[3]{x^2}\right)}{h\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{x+h}\,\sqrt[3]{x}+\sqrt[3]{x^2}\right)}$$

$$= \lim_{h\to 0}\frac{(\cancel{x}+\cancel{h})-\cancel{x}}{\cancel{h}\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{(x+h)x}+\sqrt[3]{x^2}\right)} = \frac{1}{\sqrt[3]{x^2}+\sqrt[3]{x^2}+\sqrt[3]{x^2}} = \frac{1}{3\sqrt[3]{x^2}}$$

(2) $$y' = \lim_{h\to 0}\frac{\dfrac{1}{x+h}-\dfrac{1}{x}}{h} = \lim_{h\to 0}\frac{\cancel{x}-(\cancel{x}+\cancel{h})}{\cancel{h}\cdot(x+h)x} = \lim_{h\to 0}\frac{-1}{(x+h)x} = -\frac{1}{x^2}$$

(3) $$y' = \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h} = \lim_{h\to 0}\frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$

$$= \lim_{h\to 0}\left(\frac{-\cos x(1-\cos h)}{h} - \frac{\sin x\sin h}{h}\right) = \lim_{h\to 0}\left(-2\cos x\cdot\frac{1-\cos h}{2}\cdot\frac{1}{h} - \sin x\cdot\frac{\sin h}{h}\right)$$

$$= \lim_{h\to 0}\left(-2\cos x\cdot\sin^2\frac{h}{2}\cdot\frac{h}{h^2} - \sin x\cdot\frac{\sin h}{h}\right) = \lim_{h\to 0}\left(-2\cos x\cdot\frac{\sin^2\frac{h}{2}}{\left(\frac{h}{2}\right)^2}\cdot\frac{h}{4} - \sin x\cdot\frac{\sin h}{h}\right)$$

$$= -\sin x$$

96

(1) $y' = 3x^2\left(x^2+1\right)^3 + x^3\cdot 3\left(x^2+1\right)^2\cdot 2x = 3x^2(x^2+1)^2(3x^2+1)$

(2) $y' = 2\left(x^4+2x^2+3\right)\cdot\left(4x^3+4x\right) = 8x(x^4+2x^2+3)(x^2+1)$

(3) $$y' = \frac{2(1-x)-(2x+1)(-1)}{(1-x)^2} = \frac{3}{(1-x)^2}$$

(4) $$y' = \frac{1\cdot\sqrt{x+1}-(x-1)\cdot\dfrac{1}{2\sqrt{x+1}}}{x+1} = \frac{x+3}{2(x+1)\sqrt{x+1}}$$

(5) $$y' = \frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}} = -\frac{x}{\sqrt{1-x^2}}$$

(6) $$y' = \frac{3}{\cancel{4}}(2x^2+1)^{-\frac{1}{4}}\cdot\cancel{4}x = \frac{3x}{\sqrt[4]{2x^2+1}}$$

97

(1) 両辺の対数をとると　$\log y = \log\sqrt[3]{\dfrac{x-1}{x+1}} = \dfrac{1}{3}\Big(\log(x-1)-\log(x+1)\Big)$

この両辺を x で微分すると　$\dfrac{y'}{y} = \dfrac{1}{3}\left(\dfrac{1}{x-1}-\dfrac{1}{x+1}\right) = \dfrac{2}{3(x-1)(x+1)}$

$\therefore\quad y' = \dfrac{2}{3(x-1)(x+1)}\sqrt[3]{\dfrac{x-1}{x+1}}$

(2) 両辺の対数をとると　$\log y = \log x^{\frac{1}{x}} = \dfrac{1}{x}\log x$

この両辺を x で微分すると　$\dfrac{y'}{y} = -\dfrac{1}{x^2}\log x + \dfrac{1}{x}\cdot\dfrac{1}{x} = \dfrac{1}{x^2}(1-\log x)$

$\therefore\quad y' = x^{\frac{1}{x}}\cdot\dfrac{1}{x^2}(1-\log x) = x^{\frac{1}{x}-2}(1-\log x)$

98

(1) $y' = \cos x\cdot\cos^2 x + \sin x\cdot 2\cos x(-\sin x) = \cos^3 x - 2\sin^2 x\cos x = 3\cos^3 x - 2$

(2) $$y' = \left\{\log\frac{x^2+1}{x}\right\}' = \left\{\log(x^2+1)-\log x\right\}' = \frac{2x}{x^2+1}-\frac{1}{x} = \frac{2x^2-(x^2+1)}{x(x^2+1)} = \frac{x^2-1}{x(x^2+1)}$$

(3) $$y' = \frac{(\cos x+\sin x)^2+(\sin x-\cos x)^2}{(\sin x+\cos x)^2} = \frac{\cos^2 x+\cancel{2\cos x\sin x}+\sin^2 x+\sin^2 x-\cancel{2\sin x\cos x}+\cos^2 x}{(\sin x+\cos x)^2}$$

$$= \frac{2}{(\sin x+\cos x)^2}$$

(4) $y' = \cos x e^{\sin x}$

(5) $y' = 2e^{2x}\sin^2 x + e^{2x}2\sin x\cos x = 2e^x\sin x(\sin x+\cos x)$

(6) $$y' = \Big(\log|1-\cos x| - \log|1+\cos x|\Big)' = \frac{\sin x}{1-\cos x} - \frac{-\sin x}{1+\cos x}$$

$$= \frac{\sin x+\cancel{\sin x\cos x}+\sin x-\cancel{\sin x\cos x}}{1-\cos^2 x} = \frac{2\sin x}{\sin^2 x} = \frac{2}{\sin x}$$

99

(1) $y' = \dfrac{1}{2}\left(\sqrt{x^2+1} + \dfrac{x^2}{\sqrt{x^2+1}} + \dfrac{1+\dfrac{x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}}\right)$

$= \dfrac{1}{2}\left(\sqrt{x^2+1} + \dfrac{x^2}{\sqrt{x^2+1}} + \dfrac{\cancel{\sqrt{x^2+1}+x}}{\cancel{\left(x+\sqrt{x^2+1}\right)}\sqrt{x^2+1}}\right) = \dfrac{1}{2}\left(\sqrt{x^2+1} + \dfrac{x^2+1}{\sqrt{x^2+1}}\right)$

$= \dfrac{1}{2}\left(\sqrt{x^2+1} + \sqrt{x^2+1}\right) = \sqrt{x^2+1}$

(2) $y' = \dfrac{1}{2}\left(\sqrt{1-x^2} - \dfrac{x^2}{\sqrt{1-x^2}} + \dfrac{1}{\sqrt{1-x^2}}\right) = \dfrac{1}{2}\left(\sqrt{1-x^2} + \dfrac{1-x^2}{\sqrt{1-x^2}}\right)$

$= \dfrac{1}{2}\left(\sqrt{1-x^2} + \sqrt{1-x^2}\right) = \sqrt{1-x^2}$

100

(1) ① $n=1$ のとき

$y' = -\sin x = \cos\left(x+\dfrac{\pi}{2}\right)$ だから，$n=1$ のとき成り立つ

② $n=k$ のとき成り立つとすると

$y^{(k+1)} = \{y^{(k)}\}' = \left\{\cos\left(x+\dfrac{k\pi}{2}\right)\right\}' = -\sin\left(x+\dfrac{k\pi}{2}\right) = \cos\left\{\left(x+\dfrac{k\pi}{2}\right)+\dfrac{\pi}{2}\right\}$

$= \cos\left(x+\dfrac{(k+1)\pi}{2}\right)$ だから，$n=k+1$ のときも成り立つ

以上より，数学的帰納法によって，任意の自然数 n について $y^{(n)} = \cos\left(x+\dfrac{n\pi}{2}\right)$ が成り立つ

(2) ① $n=1$ のとき

$y' = 2\cos(2x+1) = 2\sin\left(2x+1+\dfrac{\pi}{2}\right)$ だから，$n=1$ のとき成り立つ

② $n=k$ のとき成り立つとすると

$y^{(k+1)} = \{y^{(k)}\}' = \left\{2^k\sin\left(2x+1+\dfrac{k\pi}{2}\right)\right\}' = 2^k\cdot 2\cos\left(2x+1+\dfrac{k\pi}{2}\right)$

$= 2^{k+1}\sin\left\{\left(2x+1+\dfrac{k\pi}{2}\right)+\dfrac{\pi}{2}\right\} = 2^{k+1}\sin\left(2x+1+\dfrac{(k+1)\pi}{2}\right)$

だから，$n=k+1$ のときも成り立つ

以上より，数学的帰納法によって，任意の自然数 n について $y^{(n)} = \sin\left(2x+1+\dfrac{n\pi}{2}\right)$ が成り立つ

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(1) [証明]

左辺 $= (\sinh x)' = \left(\dfrac{e^x-e^{-x}}{2}\right)' = \dfrac{e^x+e^{-x}}{2} = \cosh x =$ 右辺 [証明終]

(2) [証明]

左辺 $= (\cosh x)' = \left(\dfrac{e^x+e^{-x}}{2}\right)' = \dfrac{e^x-e^{-x}}{2} = \sinh x =$ 右辺 [証明終]

(3) [証明]

左辺 $= \cosh^2 x - \sinh^2 x = \left(\dfrac{e^x+e^{-x}}{2}\right)^2 - \left(\dfrac{e^x-e^{-x}}{2}\right)^2 = \dfrac{(\cancel{e^{2x}}+2+\cancel{e^{-2x}}) - (\cancel{e^{2x}}-2+\cancel{e^{-2x}})}{4}$

$= 1 =$ 右辺 [証明終]

(4) [証明]

左辺 $= (\tanh x)' = \left(\dfrac{\sinh x}{\cosh x}\right)' = \dfrac{\cosh^2 x - \sinh^2 x}{\cosh^2 x} = \dfrac{1}{\cosh^2 x} =$ 右辺 [証明終]