新版基礎数学演習

解答

実教出版

1章　数と式

1節　整式

1 (1)　$3x^3+xy^3-2x^2y+4y^2$ ［x］

$\Longrightarrow 3x^3-2yx^2+y^3x+4y^2$

整式の次数は **4 次**，x の次数は **3 次**，x^3 の係数は **3**，x^2 の係数は $\boldsymbol{-2y}$，x の係数は $\boldsymbol{y^3}$，定数項は $\boldsymbol{4y^2}$

(2)　$-3a^2b+5b^2+ab+2a^2b^3-4ab^2$ ［a］

$\Longrightarrow (-3b+2b^3)a^2+(b-4b^2)a+5b^2$

整式の次数は **5 次**，a の次数は **2 次**，a^2 の係数は $\boldsymbol{-3b+2b^3}$

a の係数は $\boldsymbol{b-4b^2}$　定数項は $\boldsymbol{5b^2}$

2　$A=x^2+10x-8$，$B=2x^2-5$，$C=7x-1$

(1)　$A+B$

$=(x^2+10x-8)+(2x^2-5)$

$=\boldsymbol{3x^2+10x-13}$

(2)　$A-C$

$=(x^2+10x-8)-(7x-1)$

$=\boldsymbol{x^2+3x-7}$

(3)　$A-(B-C)$

$=(x^2+10x-8)$

$\qquad -\{(2x^2-5)-(7x-1)\}$

$=x^2+10x-8-(2x^2-7x-4)$

$=\boldsymbol{-x^2+17x-4}$

(4)　$A-(2B+C)$

$=x^2+10x-8-\{2(2x^2-5)+(7x-1)\}$

$=x^2+10x-8-(4x^2-10+7x-1)$

$=x^2+10x-8-4x^2-7x+11$

$=\boldsymbol{-3x^2+3x+3}$

3 (1)　$a^5\times a^3=\boldsymbol{a^8}$

(2)　$2xy^3\times(-3xy)=\boldsymbol{-6x^2y^4}$

(3)　$(-x)^3\times(-x^2)=\boldsymbol{x^5}$

(4)　$-ab\times(-a^3b)=\boldsymbol{a^4b^2}$

(5)　$4ab^2\times\left(-\dfrac{1}{2}ab\right)^2$

$=4ab^2\times\dfrac{1}{4}a^2b^2=\boldsymbol{a^3b^4}$

(6)　$ax^2\times(-y)^3\times(-bxy)$

$=-ax^2y^3\times(-bxy)$

$=\boldsymbol{abx^3y^4}$

4 (1)　$xy(x-y)=\boldsymbol{x^2y-xy^2}$

(2)　$(-ab)^2(a-b+1)$

$=a^2b^2(a-b+1)$

$=\boldsymbol{a^3b^2-a^2b^3+a^2b^2}$

(3)　$(2x+y)(x-y+1)$

$=2x^2-2xy+2x+xy-y^2+y$

$=\boldsymbol{2x^2-xy+2x+y-y^2}$

(4)　$(a^2-2ab+2b^2)(a-3b)$

$=a^3-3a^2b-2a^2b+6ab^2+2ab^2-6b^3$

$=\boldsymbol{a^3-5a^2b+8ab^2-6b^3}$

(5)　$(x^2+2x-1)(x^2+x+3)$

$=x^4+x^3+3x^2+2x^3+2x^2+6x$

$\qquad -x^2-x-3$

$=\boldsymbol{x^4+3x^3+4x^2+5x-3}$

(6)　$(4x^3+x-3)(x^2+2)$

$=4x^5+8x^3+x^3+2x-3x^2-6$

$=\boldsymbol{4x^5+9x^3-3x^2+2x-6}$

(7)　$(x-3)(2x^3+4x-1)$

$=2x^4+4x^2-x-6x^3-12x+3$

$=\boldsymbol{2x^4-6x^3+4x^2-13x+3}$

(8)　$(2x^3+x^2-3x-5)(x^2-x+4)$

$=2x^5-2x^4+8x^3+x^4-x^3+4x^2$

$\qquad -3x^3+3x^2-12x-5x^2+5x-20$

$=\boldsymbol{2x^5-x^4+4x^3+2x^2-7x-20}$

5 (1)　$(2x+1)^2$

$=\boldsymbol{4x^2+4x+1}$

(2)　$(x-6)(x+6)$

$= \boldsymbol{x^2-36}$

(3)　$(x-3)(x+4)$

$= \boldsymbol{x^2+x-12}$

(4)　$(3x-2)(4x+5)$

$= 12x^2+15x-8x-10$

$= \boldsymbol{12x^2+7x-10}$

(5)　$(2x+5)^3$

$= 8x^3+3\cdot 4x^2\cdot 5+3\cdot 2x\cdot 25+125$

$= \boldsymbol{8x^3+60x^2+150x+125}$

(6)　$(3x-4)^3$

$= 27x^3-3\cdot 9x^2\cdot 4+3\cdot 3x\cdot 16-64$

$= \boldsymbol{27x^3-108x^2+144x-64}$

6 (1)　$a+b=X$　とおくと

$(a+b+1)(a+b-1)$

$= (X+1)(X-1)$

$= X^2-1$

$= (a+b)^2-1$

$= \boldsymbol{a^2+2ab+b^2-1}$

(2)　$a+1=X$　とおくと

$(a+2a+1)(a-3a+1)$

$= (X+2a)(X-3a)$

$= X^2-aX-6a^2$

$= (a+1)^2-a(a+1)-6a^2$

$= a^2+2a+1-a^2-a-6a^2$

$= \boldsymbol{-6a^2+a+1}$

(3)　$x+y=X$　とおくと

$(x+y-1)^2$

$= (X-1)^2$

$= X^2-2X+1$

$= (x+y)^2-2(x+y)+1$

$= \boldsymbol{x^2+2xy+y^2-2x-2y+1}$

(4)　$(x-5)^2(x+5)^2$

$= \{(x-5)(x+5)\}^2$

$= (x^2-25)^2$

$= \boldsymbol{x^4-50x^2+625}$

(5)　$(x-1)(x+1)(x^2+1)$

$= (x^2-1)(x^2+1)$

$= \boldsymbol{x^4-1}$

(6)　$(2x-3y)(2x+3y)(4x^2+9y^2)$

$= (4x^2-9y^2)(4x^2+9y^2)$

$= \boldsymbol{16x^4-81y^4}$

7 (1)　$x^3y-x^2y^2-12xy^3$

$= xy(x^2-xy-12y^2)$

$= \boldsymbol{xy(x-4y)(x+3y)}$

(2)　$x(a-b)-3(b-a)$

$= \boldsymbol{(a-b)(x+3)}$

(3)　$a^2+12a+36$

$= \boldsymbol{(a+6)^2}$

(4)　$4x^2+12xy+9y^2$

$= \boldsymbol{(2x+3y)^2}$

(5)　x^2-64

$= \boldsymbol{(x+8)(x-8)}$

(6)　$x^2+2x-24$

$= \boldsymbol{(x+6)(x-4)}$

(7)　$x^2-3x-18$

$= \boldsymbol{(x-6)(x+3)}$

(8)　$x^2+8xy+12y^2$

$= \boldsymbol{(x+2y)(x+6y)}$

8 (1)　$3x^2-x-10$

$= \boldsymbol{(3x+5)(x-2)}$

$$\begin{array}{ccccc} 3 & \times & 5 & \longrightarrow & 5 \\ 1 & & -2 & \longrightarrow & -6 \\ \hline 3 & & -10 & & -1 \end{array}$$

(2)　$10x^2-23x+12$

$= \boldsymbol{(5x-4)(2x-3)}$

$$\begin{array}{ccccc} 5 & \times & -4 & \longrightarrow & -8 \\ 2 & & -3 & \longrightarrow & -15 \\ \hline 10 & & 12 & & -23 \end{array}$$

(3)　$12x^2+4xy-y^2$

$= \boldsymbol{(2x+y)(6x-y)}$

$$\begin{array}{ccccc} 2 & \times & 1 & \longrightarrow & 6 \\ 6 & & -1 & \longrightarrow & -2 \\ \hline 12 & & -1 & & 4 \end{array}$$

(4)　$6x^2+xy-15y^2$

$= \boldsymbol{(3x+5y)(2x-3y)}$

$$\begin{array}{ccccc} 3 & \times & 5 & \longrightarrow & 10 \\ 2 & & -3 & \longrightarrow & -9 \\ \hline 6 & & -15 & & 1 \end{array}$$

(5)　$4m^2-5mn-6n^2$

$= \boldsymbol{(4m+3n)(m-2n)}$

$$\begin{array}{ccc} 4 & 3 \longrightarrow & 3 \\ 1 & -2 \longrightarrow & -8 \\ \hline 4 & -6 & -5 \end{array}$$

(6) $4ab^2-14ab+6a$

$=2a(2b^2-7b+3)$

$=\mathbf{2a(2b-1)(b-3)}$

$$\begin{array}{ccc} 2 & -1 \longrightarrow & -1 \\ 1 & -3 \longrightarrow & -6 \\ \hline 2 & 3 & -7 \end{array}$$

9 (1) a^3-64

$=a^3-4^3$

$=\mathbf{(a-4)(a^2+4a+16)}$

(2) $27a^3+125=(3a)^3+5^3$

$=\mathbf{(3a+5)(9a^2-15a+25)}$

(3) $24x^3-81y^3$

$=3(8x^3-27y^3)=3\{(2x)^3-(3y)^3\}$

$=\mathbf{3(2x-3y)(4x^2+6xy+9y^2)}$

10 (1) $a-b=X$ とおくと

$(a-b)^2-6(a-b)+9$

$=X^2-6X+9$

$=(X-3)^2$

$=\mathbf{(a-b-3)^2}$

(2) $x+y=X$ とおくと

$(x+y)^2+5(x+y)-14$

$=X^2+5X-14$

$=(X+7)(X-2)$

$=\mathbf{(x+y+7)(x+y-2)}$

(3) $x+2=X$, $y-2=Y$ とおくと

$(x+2)^2-(y-2)^2$

$=X^2-Y^2$

$=(X+Y)(X-Y)$

$=(x+2+y-2)(x+2-y+2)$

$=\mathbf{(x+y)(x-y+4)}$

(4) $x-2y=X$, $2x-y=Y$ とおくと

$(x-2y)^2-(2x-y)^2$

$=X^2-Y^2$

$=(X+Y)(X-Y)$

$=(x-2y+2x-y)(x-2y-2x+y)$

$=(3x-3y)(-x-y)$

$=\mathbf{-3(x-y)(x+y)}$

(5) $ab+ac+b^2+bc$

$=a(b+c)+b(b+c)$

$=\mathbf{(a+b)(b+c)}$

(6) $a^2b+a^2c+ab^2-b^2c$

$=ab(a+b)+c(a^2-b^2)$

$=ab(a+b)+c(a+b)(a-b)$

$=\mathbf{(a+b)(ab-bc+ca)}$

11 (1) $ax^2-(a+1)x+1$

$=\mathbf{(ax-1)(x-1)}$

(2) $abx^2-(a-b)x-1$

$=\mathbf{(ax+1)(bx-1)}$

(3) $b+c=X$ とおくと

$a^2-2a(b+c)+(b+c)^2$

$=a^2-2aX+X^2$

$=(a-X)^2$

$=\mathbf{(a-b-c)^2}$

(4) $x^2+2x-(y+1)(y-1)$

$=\{x+(y+1)\}\{x-(y-1)\}$

$=\mathbf{(x+y+1)(x-y+1)}$

$$\begin{array}{ccc} x & y+1 \longrightarrow & x(y+1) \\ x & -(y-1) \longrightarrow & -x(y-1) \\ \hline x^2 & -(y+1)(y-1) & 2x \end{array}$$

(5) $x^2+3xy+2y^2-5x-8y+6$

$=x^2+(3y-5)x+(2y^2-8y+6)$

$=x^2+(3y-5)x+(2y-2)(y-3)$

$=\{x+(2y-2)\}\{x+(y-3)\}$

$=\mathbf{(x+2y-2)(x+y-3)}$

$$\begin{array}{ccc} x & 2y-2 \longrightarrow & x(2y-2) \\ x & y-3 \longrightarrow & x(y-3) \\ \hline x^2 & (2y-2)(y-3) & (3y-5)x \end{array}$$

(6) $2x^2+7x+3xy-2y^2-y+3$

$=2x^2+(3y+7)x-2y^2-y+3$

$=2x^2+(3y+7)x-(y-1)(2y+3)$

$=\{2x-(y-1)\}\{x+(2y+3)\}$

$=\mathbf{(2x-y+1)(x+2y+3)}$

$$\begin{array}{ccc} 2x & -(y-1) \longrightarrow & -x(y-1) \\ x & 2y+3 \longrightarrow & 2x(2y+3) \\ \hline 2x^2 & -(y-1)(2y+3) & (3y+7)x \end{array}$$

12 (1) $a^2+a+ac-b^2-b+bc+c$

$= a^2 + (1+c)a - (b+1)(b-c)$
$= \{a + (b+1)\}\{a - (b-c)\}$
$= \boldsymbol{(a+b+1)(a-b+c)}$

$$\begin{array}{lcl} a & b+1 \longrightarrow & a(b+1) \\ a & -(b-c) \longrightarrow & -a(b-c) \\ \hline a^2 & -(b+1)(b-c) & (1+c)a \end{array}$$

(2)　$(a+b)(b+c)(c+a) + abc$
$= (ab + ac + b^2 + bc)(c+a) + abc$
$= abc + a^2b + ac^2 + a^2c + b^2c$
$\qquad + ab^2 + bc^2 + abc + abc$
$= (b+c)a^2 + (b^2 + c^2 + 3bc)a$
$\qquad + bc(b+c)$
$= \{a + (b+c)\}\{(b+c)a + bc\}$
$= \boldsymbol{(a+b+c)(ab+bc+ca)}$

$$\begin{array}{lcl} a & b+c \longrightarrow & a(b+c)^2 \\ (b+c)a & bc \longrightarrow & abc \\ \hline (b+c)a^2 & bc(b+c) & (b^2+c^2+3bc)a \end{array}$$

(3)　$x^4 + 64$
$= (x^4 + 16x^2 + 64) - 16x^2$
$= (x^2+8)^2 - (4x)^2$
$= \boldsymbol{(x^2+4x+8)(x^2-4x+8)}$

(4)　$x^4 - 8x^2y^2 + 4y^4$
$= (x^4 - 4x^2y^2 + 4y^4) - 4x^2y^2$
$= (x^2 - 2y^2)^2 - (2xy)^2$
$= \boldsymbol{(x^2 - 2y^2 + 2xy)(x^2 - 2y^2 - 2xy)}$

13　(1)　$2^n + 2^n = 2^n(1+1) = 2 \cdot 2^n = \boldsymbol{2^{n+1}}$

(2)　$3^{n+1} - 3^n$
$= 3^n(3-1) = \boldsymbol{2 \cdot 3^n}$

(3)　$18 \times 3^n + 3^{n+2} = 3^n(18 + 3^2)$
$= 27 \cdot 3^n = 3^3 \cdot 3^n = \boldsymbol{3^{n+3}}$

14　ある多項式を A とおくと，
$A - (2x^2 - 3xy + y^2) = x^2 + xy - 5y^2$
$A = x^2 + xy - 5y^2 + (2x^2 - 3xy + y^2)$
$\quad = 3x^2 - 2xy - 4y^2$

したがって，正しい答えは
$3x^2 - 2xy - 4y^2 + (2x^2 - 3xy + y^2)$
$= \boldsymbol{5x^2 - 5xy - 3y^2}$

15　(1)　$(a+b)(a-b)(a^2+ab+b^2)$
$\qquad (a^2 - ab + b^2)$
$= (a+b)(a^2 - ab + b^2)$
$\qquad (a-b)(a^2+ab+b^2)$
$= (a^3 + b^3)(a^3 - b^3)$
$= \boldsymbol{a^6 - b^6}$

(2)　$(x+2)(x-3)(x^2-2x+4)$
$\qquad (x^2+3x+9)$
$= (x+2)(x^2-2x+4)(x-3)$
$\qquad (x^2+3x+9)$
$= (x^3+8)(x^3-27) = \boldsymbol{x^6 - 19x^3 - 216}$

(3)　$(2a+b)(a-3b)(2a-b)(a+3b)$
$= (2a+b)(2a-b)(a-3b)(a+3b)$
$= (4a^2 - b^2)(a^2 - 9b^2)$
$= \boldsymbol{4a^4 - 37a^2b^2 + 9b^4}$

16　(1)　$(x+1)(x-3)(x^2-3)$
$= (x^2 - 2x - 3)(x^2 - 3)$
$= x^4 - 3x^2 - 2x^3 + 6x - 3x^2 + 9$
$= \boldsymbol{x^4 - 2x^3 - 6x^2 + 6x + 9}$

(2)　$(x+1)(x-2)(x+3)(x-4)$
$= (x^2 - x - 2)(x^2 - x - 12)$
$x^2 - x = X$ とおくと
$= (X-2)(X-12)$
$= X^2 - 14X + 24$
$= (x^2 - x)^2 - 14(x^2 - x) + 24$
$= x^4 - 2x^3 + x^2 - 14x^2 + 14x + 24$
$= \boldsymbol{x^4 - 2x^3 - 13x^2 + 14x + 24}$

(3)　$(x+1)(x-2)(x+4)(x-8)$
$= (x+1)(x-8)(x-2)(x+4)$
$= (x^2 - 7x - 8)(x^2 + 2x - 8)$
$x^2 - 8 = X$ とおくと
$= (X - 7x)(X + 2x)$
$= X^2 - 5Xx - 14x^2$
$= (x^2 - 8)^2 - 5(x^2 - 8)x - 14x^2$
$= x^4 - 16x^2 + 64 - 5x^3 + 40x - 14x^2$
$= \boldsymbol{x^4 - 5x^3 - 30x^2 + 40x + 64}$

(4)　$(x^2 + x + 1)(2x^2 + 2x - 3)$
$= (x^2 + x + 1)\{2(x^2 + x) - 3\}$
$x^2 + x = X$ とおくと
$= (X+1)(2X-3)$
$= 2X^2 - X - 3$
$= 2(x^2 + x)^2 - (x^2 + x) - 3$
$= 2(x^4 + 2x^3 + x^2) - x^2 - x - 3$
$= \boldsymbol{2x^4 + 4x^3 + x^2 - x - 3}$

17　(1)　$(a+b+c)^2 - (a+b-c)^2$

$+(a-b+c)^2-(a-b-c)^2$

$=\{(a+b)+c\}^2-\{(a+b)-c\}^2$

$+\{(a-b)+c\}^2-\{(a-b)-c\}^2$

$=(a+b)^2+2c(a+b)+c^2$

$-(a+b)^2+2c(a+b)-c^2$

$+(a-b)^2+2c(a-b)+c^2$

$-(a-b)^2+2c(a-b)-c^2$

$=4c(a+b)+4c(a-b)$

$=\boldsymbol{8ac}$

(2) $(a+b+c)^2-(b+c-a)^2$

$+(c+a-b)^2-(c-b-a)^2$

$=\{(b+c)+a\}^2-\{(b+c)-a\}^2$

$+\{(c-b)+a\}^2-\{(c-b)-a\}^2$

$=(b+c)^2+2a(b+c)+a^2$

$-(b+c)^2+2a(b+c)-a^2$

$+(c-b)^2+2a(c-b)+a^2$

$-(c-b)^2+2a(c-b)-a^2$

$=4a(b+c)+4a(c-b)$

$=\boldsymbol{8ac}$

18 (1) $a^5-16a=a(a^4-16)$

$=a(a^2+4)(a^2-4)$

$=\boldsymbol{a(a^2+4)(a+2)(a-2)}$

(2) $81a^4-72a^2b^2+16b^4$

$=(9a^2-4b^2)^2$

$=\boldsymbol{(3a+2b)^2(3a-2b)^2}$

(3) $x^4-x^2y^2-12y^4$

$=(x^2-4y^2)(x^2+3y^2)$

$=\boldsymbol{(x+2y)(x-2y)(x^2+3y^2)}$

(4) $4x^4-37x^2y^2+9y^4$

$=(4x^2-y^2)(x^2-9y^2)$

$=\boldsymbol{(2x+y)(2x-y)(x+3y)(x-3y)}$

19 (1) $x^3+6x^2+12x+8$

$=x^3+3x^2\cdot 2+3\cdot x\cdot 2^2+2^3$

$=\boldsymbol{(x+2)^3}$

(2) $27x^3+27x^2+9x+1$

$=(3x)^3+3\cdot(3x)^2\cdot 1+3\cdot 3x\cdot 1+1$

$=\boldsymbol{(3x+1)^3}$

(3) x^6-64y^6

$=(x^3)^2-\{(2y)^3\}^2$

$=\{x^3+(2y)^3\}\{x^3-(2y)^3\}$

$=\boldsymbol{(x+2y)(x^2-2xy+4y^2)}$

$\boldsymbol{\times(x-2y)(x^2+2xy+4y^2)}$

(4) $64x^6-48x^4+12x^2-1$

$=(4x^2)^3-3\cdot(4x)^2\cdot 1+3\cdot 4x^2\cdot 1-1$

$=(4x^2-1)^3=\boldsymbol{(2x+1)^3(2x-1)^3}$

20 (1) $(a-b-c+d)(a+b-c-d)$

$=\{(a-c)-(b-d)\}$

$\{(a-c)+(b-d)\}$

$=(a-c)^2-(b-d)^2$

$=a^2-2ac+c^2-(b^2-2bd+d^2)$

$=\boldsymbol{a^2-b^2+c^2-d^2-2ac+2bd}$

(2) $a^2+1=X$ とおくと

$(a^8-a^4+1)(a^4-a^2+1)(a^2-a+1)$

(a^2+a+1)

$=(a^8-a^4+1)(a^4-a^2+1)$

$(X-a)(X+a)$

$=(a^8-a^4+1)(a^4-a^2+1)(X^2-a^2)$

$=(a^8-a^4+1)(a^4-a^2+1)$

$\{(a^2+1)^2-a^2\}$

$=(a^8-a^4+1)(a^4-a^2+1)(a^4+a^2+1)$

$a^4+1=Y$ とおくと

$=(a^8-a^4+1)(Y-a^2)(Y+a^2)$

$=(a^8-a^4+1)(Y^2-a^4)$

$=(a^8-a^4+1)\{(a^4+1)^2-a^4\}$

$=(a^8-a^4+1)(a^8+a^4+1)$

$a^8+1=Z$ とおくと

$=(Z-a^4)(Z+a^4)$

$=Z^2-a^8$

$=(a^8+1)^2-a^8$

$=\boldsymbol{a^{16}+a^8+1}$

21 (1) $(xy+1)(x+1)(y+1)+xy$

$=(xy+1)(xy+x+y+1)+xy$

$A=xy+1$ とおくと

$=A(A+x+y)+xy$

$=A^2+(x+y)A+xy$

$=(A+x)(A+y)$

$=\boldsymbol{(xy+1+x)(xy+1+y)}$

(2) $(b-c)^3+(c-a)^3+(a-b)^3$

$=\{(b-c)+(c-a)\}$

$\{(b-c)^2-(b-c)(c-a)+(c-a)^2\}$

$+(a-b)^3$

$=(b-a)\{(b-c)^2-(b-c)(c-a)$

$+(c-a)^2-(a-b)^2\}$

$= (b-a)$
$\{(b^2-2bc+c^2)-(bc-ab-c^2+ca)$
$+(c^2-2ca+a^2)-(a^2-2ab+b^2)\}$
$= (b-a)(3c^2-3bc-3ca+3ab)$
$= 3(b-a)\{c^2-(a+b)c+ab\}$
$= 3(b-a)(c-a)(c-b)$
$= \boldsymbol{3(a-b)(b-c)(c-a)}$

(3)　$a^3+a^2-(b^2+1)a+b^2-1$
$= a^3+a^2-ab^2-a+b^2-1$
$= a^3-1+a^2-a-b^2(a-1)$
$= (a-1)(a^2+a+1)+a(a-1)$
$-b^2(a-1)$
$= (a-1)(a^2+a+1+a-b^2)$
$= (a-1)\{(a+1)^2-b^2\}$
$= \boldsymbol{(a-1)(a+1+b)(a+1-b)}$

22　$(x+1)(x+2)(x+3)(x+4)-24$
$= \{(x+1)(x+4)\}\{(x+2)(x+3)\}-24$
$= (x^2+5x+4)(x^2+5x+6)-24$
$x^2+5x=X$ とおくと
$= (X+4)(X+6)-24$
$= X^2+10X+24-24$
$= X(X+10)$
$= (x^2+5x)(x^2+5x+10)$
$= \boldsymbol{x(x+5)(x^2+5x+10)}$

23　(1)　$(x+y)^3-3xy(x+y)$
$= x^3+3x^2y+3xy^2+y^3-3x^2y$
$-3xy^2$
$= \boldsymbol{x^3+y^3}$

(2)　$x^3+y^3+z^3-3xyz$
$= (x+y)^3-3xy(x+y)+z^3-3xyz$
$= (x+y)^3+z^3-3xy(x+y+z)$
$= \{(x+y)+z\}$
$\times\{(x+y)^2-(x+y)z+z^2\}$
$-3xy(x+y+z)$
$= (x+y+z)$
$\times(x^2+2xy+y^2-xz-yz+z^2-3xy)$
$= \boldsymbol{(x+y+z)}$
$\boldsymbol{\times(x^2+y^2+z^2-xy-yz-zx)}$

2節　整式の除法と分数式

24　(1)

$$\begin{array}{r@{\,}r} & 2x-1 \\ x-1\big) & \overline{2x^2-3x+5} \\ & 2x^2-2x \\ \hline & -x+5 \\ & -x+1 \\ \hline & 4 \end{array}$$

商 **$\boldsymbol{2x-1}$，** 余り **4**

(2)

$$\begin{array}{r@{\,}r} & x+1 \\ 3x+1\big) & \overline{3x^2+4x-6} \\ & 3x^2+x \\ \hline & 3x-6 \\ & 3x+1 \\ \hline & -7 \end{array}$$

商 **$\boldsymbol{x+1}$，** 余り **$\boldsymbol{-7}$**

(3)

$$\begin{array}{r@{\,}r} & 2x^2+x+1 \\ 2x-1\big) & \overline{4x^3 \qquad +x-1} \\ & 4x^3-2x^2 \\ \hline & 2x^2+x-1 \\ & 2x^2-x \\ \hline & 2x-1 \\ & 2x-1 \\ \hline & 0 \end{array}$$

商 **$\boldsymbol{2x^2+x+1}$，** 余り **0**

(4)

$$\begin{array}{r@{\,}r} & x^2-2x+1 \\ 4x+3\big) & \overline{4x^3-5x^2-2x+3} \\ & 4x^3+3x^2 \\ \hline & -8x^2-2x+3 \\ & -8x^2-6x \\ \hline & 4x+3 \\ & 4x+3 \\ \hline & 0 \end{array}$$

商 **$\boldsymbol{x^2-2x+1}$，** 余り **0**

25　(1)

$$\begin{array}{r@{\,}r} & 3x+4 \\ x^2-2x-2\big) & \overline{3x^3-2x^2+x-1} \\ & 3x^3-6x^2-6x \\ \hline & 4x^2+7x-1 \\ & 4x^2-8x-8 \\ \hline & 15x+7 \end{array}$$

商 **$\boldsymbol{3x+4}$，** 余り **$\boldsymbol{15x+7}$**

(2)
$$\begin{array}{r} 2x-3 \\ x^2+2x+3\,\overline{\smash{)}\,2x^3+x^2-13x-7} \\ \underline{2x^3+4x^2+6x} \\ -3x^2-19x-7 \\ \underline{-3x^2-6x-9} \\ -13x+2 \end{array}$$

商 $\boldsymbol{2x-3}$，余り $\boldsymbol{-13x+2}$

(3)
$$\begin{array}{r} x-2 \\ 2x^2+4x-3\,\overline{\smash{)}\,2x^3-8x+7} \\ \underline{2x^3+4x^2-3x} \\ -4x^2-5x+7 \\ \underline{-4x^2-8x+6} \\ 3x+1 \end{array}$$

商 $\boldsymbol{x-2}$，余り $\boldsymbol{3x+1}$

(4)
$$\begin{array}{r} 2x-1 \\ 2x^2+5\,\overline{\smash{)}\,4x^3-2x^2+5} \\ \underline{4x^3+10x} \\ -2x^2-10x+5 \\ \underline{-2x^2-5} \\ -10x+10 \end{array}$$

商 $\boldsymbol{2x-1}$，余り $\boldsymbol{-10x+10}$

26 $x^3+4x^2-9=A(x+3)$ より

$A=(x^3+4x^2-9)\div(x+3)$

$$\begin{array}{r} x^2+x-3 \\ x+3\,\overline{\smash{)}\,x^3+4x^2-9} \\ \underline{x^3+3x^2} \\ x^2-9 \\ \underline{x^2+3x} \\ -3x-9 \\ \underline{-3x-9} \\ 0 \end{array}$$

よって　$A=\boldsymbol{x^2+x-3}$

27 $x^3+x^2+7=A(x^2-x+2)+3$ より

$A=(x^3+x^2+4)\div(x^2-x+2)$

$$\begin{array}{r} x+2 \\ x^2-x+2\,\overline{\smash{)}\,x^3+x^2+4} \\ \underline{x^3-x^2+2x} \\ 2x^2-2x+4 \\ \underline{2x^2-2x+4} \\ 0 \end{array}$$

よって　$A=\boldsymbol{x+2}$

28 (1)　最大公約数　$\boldsymbol{xy}$

最小公倍数　$\boldsymbol{x^2y^2z^2}$

(2)　$2x^2+3x+1=(2x+1)(x+1)$

$4x^2-1=(2x-1)(2x+1)$

よって　最大公約数　$\boldsymbol{2x+1}$

最小公倍数　$\boldsymbol{(2x+1)(2x-1)(x+1)}$

29 (1)　$\dfrac{27x^2y}{-3xy}=\dfrac{9x}{-1}$

$=\boldsymbol{-9x}$

(2)　$\dfrac{8a^3b^4}{(-2ab)^2}=\dfrac{8a^3b^4}{4a^2b^2}$

$=\boldsymbol{2ab^2}$

(3)　$\dfrac{(-2x^3y)^3}{(x^2y)^2}=\dfrac{-8x^9y^3}{x^4y^2}$

$=\boldsymbol{-8x^5y}$

(4)　$\dfrac{(6a^2b^3)^2}{(-3ab)^3}=\dfrac{36a^4b^6}{-27a^3b^3}$

$=\boldsymbol{-\dfrac{4}{3}ab^3}$

30 (1)　$\dfrac{x-1}{x+2}\times\dfrac{x^2+2x}{x^2+2x-3}$

$=\dfrac{\cancel{x-1}}{\cancel{x+2}}\times\dfrac{x(\cancel{x+2})}{(\cancel{x-1})(x+3)}$

$=\boldsymbol{\dfrac{x}{x+3}}$

(2)　$\dfrac{x^2-4}{x^2-3x+2}\times\dfrac{x^2-1}{x^2+3x+2}$

$=\dfrac{(\cancel{x+2})(\cancel{x-2})}{(\cancel{x-1})(\cancel{x-2})}\times\dfrac{(\cancel{x+1})(\cancel{x-1})}{(\cancel{x+1})(\cancel{x+2})}$

$=\boldsymbol{1}$

(3)　$\dfrac{(-2xy)^3}{a^2b^3}\div\dfrac{(xy)^2}{(-ab)^2}$

$=\dfrac{-8x^{\cancel{3}}y^{\cancel{3}}}{\cancel{a^2}b^{\cancel{3}}}\times\dfrac{\cancel{a^2}\cancel{b^2}}{\cancel{x^2}\cancel{y^2}}$

$=\boldsymbol{\dfrac{-8xy}{b}}$

(4)　$\dfrac{x^2-3x}{x^2+6x+5}\div\dfrac{x^2-6x+9}{x+5}$

$=\dfrac{x(\cancel{x-3})}{(x+1)(\cancel{x+5})}\times\dfrac{\cancel{x+5}}{(x-3)^{\cancel{2}}}$

$=\dfrac{x}{(x+1)(x-3)}$

$=\boldsymbol{\dfrac{x}{x^2-2x-3}}$

31 (1)　$\dfrac{x^2}{x-2}-\dfrac{4}{x-2}$

$$= \frac{x^2-4}{x-2}$$

$$= \frac{(x-2)(x+2)}{x-2}$$

$$= \boldsymbol{x+2}$$

(2) $\dfrac{3}{x-1} - \dfrac{x-4}{1-x}$

$$= \frac{3}{x-1} + \frac{x-4}{x-1}$$

$$= \frac{x-1}{x-1}$$

$$= \mathbf{1}$$

(3) $\dfrac{x}{x+3} + \dfrac{2}{x-1}$

$$= \frac{x(x-1)+2(x+3)}{(x+3)(x-1)}$$

$$= \frac{x^2-x+2x+6}{(x+3)(x-1)}$$

$$= \boldsymbol{\frac{x^2+x+6}{(x+3)(x-1)}}$$

(4) $\dfrac{a-2b}{ab-b^2} - \dfrac{b}{ab-a^2}$

$$= \frac{a-2b}{b(a-b)} - \frac{b}{a(b-a)}$$

$$= \frac{a-2b}{b(a-b)} + \frac{b}{a(a-b)}$$

$$= \frac{a(a-2b)+b^2}{ab(a-b)}$$

$$= \frac{a^2-2ab+b^2}{ab(a-b)}$$

$$= \frac{(a-b)^{\cancel{2}}}{ab\cancel{(a-b)}}$$

$$= \boldsymbol{\frac{a-b}{ab}}$$

32 (1) $\dfrac{1}{1-\dfrac{x}{x-1}}$

$$= \frac{1}{1-\dfrac{x}{x+1}} \times \frac{x+1}{x+1}$$

$$= \frac{x+1}{x+1-x} = \boldsymbol{x+1}$$

(2) $\dfrac{1-\dfrac{1}{x}}{x-\dfrac{1}{x}} = \dfrac{1-\dfrac{1}{x}}{x-\dfrac{1}{x}} \times \dfrac{x}{x}$

$$= \frac{x-1}{x^2-1} = \frac{x-1}{(x+1)(x-1)}$$

$$= \boldsymbol{\frac{1}{x+1}}$$

33 (1)

$$\begin{array}{r|l}
 & x-3a \\ \hline
x+a & x^2-2ax-3a^2 \\
 & x^2+ax \\ \hline
 & \quad -3ax-3a^2 \\
 & \quad -3ax-3a^2 \\ \hline
 & \qquad\qquad 0
\end{array}$$

商　$\boldsymbol{x-3a}$，余り　$\mathbf{0}$

(2)

$$\begin{array}{r|l}
 & x^2-2xy+2y^2 \\ \hline
x+y & x^3-x^2y \qquad\quad +2y^3 \\
 & x^3+x^2y \\ \hline
 & \quad -2x^2y \qquad\quad +2y^3 \\
 & \quad -2x^2y-2xy^2 \\ \hline
 & \qquad\qquad 2xy^2+2y^3 \\
 & \qquad\qquad 2xy^2+2y^3 \\ \hline
 & \qquad\qquad\qquad 0
\end{array}$$

商　$\boldsymbol{x^2-2xy+2y^2}$，余り　$\mathbf{0}$

(3)

$$\begin{array}{r|l}
 & x^2+2xy-2y^2 \\ \hline
x-2y & x^3 \qquad\quad -6xy^2+4y^3 \\
 & x^3-2x^2y \\ \hline
 & \quad 2x^2y-6xy^2+4y^3 \\
 & \quad 2x^2y-4xy^2 \\ \hline
 & \qquad\quad -2xy^2+4y^3 \\
 & \qquad\quad -2xy^2+4y^3 \\ \hline
 & \qquad\qquad\qquad 0
\end{array}$$

商　$\boldsymbol{x^2+2xy-2y^2}$，余り　$\mathbf{0}$

(4)

$$\begin{array}{r|l}
 & x-y \\ \hline
x^2+2xy+3y^2 & x^3+x^2y+xy^2-3y^3 \\
 & x^3+2x^2y+3xy^2 \\ \hline
 & \quad -x^2y-2xy^2-3y^3 \\
 & \quad -x^2y-2xy^2-3y^3 \\ \hline
 & \qquad\qquad\qquad 0
\end{array}$$

商　$\boldsymbol{x-y}$，余り　$\mathbf{0}$

34 $P = Q(x-1)+1$

$Q = (x^2+1)(x+1)+x-2$　より

$$P = \{(x^2+1)(x+1)+(x-2)\}(x-1)+1$$

$$= (x^2+1)(x^2-1)+(x-2)(x-1)+1$$

$$= (x^4-1)+x^2-3x+2+1$$

$$= \boldsymbol{x^4+x^2-3x+2}$$

35

$$
\begin{array}{r|l}
 & x^2+(a-2)x+(b-2a+4) \\ \hline
x+2 & x^3+ax^2+bx-6 \\
 & x^3+2x^2 \\ \hline
 & (a-2)x^2+bx-6 \\
 & (a-2)x^2+2(a-2)x \\ \hline
 & \{b-2(a-2)\}x-6 \\
 & (b-2a+4)x+2(b-2a+4) \\ \hline
 & -6-2(b-2a+4)
\end{array}
$$

割り切れるので，余りは 0，したがって

$-6-2(b-2a+4)=0$

$-6-2b+4a-8=0$

$4a-2b=14$

$2a-b=7 \quad \cdots①$

$$
\begin{array}{r|l}
 & x^2+(a+1)x+(b+a+1) \\ \hline
x-1 & x^3+ax^2+bx-6 \\
 & x^3-x^2 \\ \hline
 & (a+1)x^2+bx-6 \\
 & (a+1)x^2-(a+1)x \\ \hline
 & \{b+(a+1)\}x-6 \\
 & (b+a+1)x-(b+a+1) \\ \hline
 & -6+(b+a+1)
\end{array}
$$

割り切れるので，余りは 0，したがって

$-6+b+a+1=0$

$a+b=5 \quad \cdots②$

①，②より

$$
\begin{array}{rrcl}
 & 2a-b & = & 7 \\
+) & a+b & = & 5 \\ \hline
 & 3a & = & 12
\end{array}
$$

よって $a=4$，②に代入して

$4+b=5$ より $b=1$

したがって　$\boldsymbol{a=4,\ b=1}$

36 (1) $\dfrac{x^2+x-2}{x^2+7x+12} \div \dfrac{x^2-2x+1}{x^2-x-12} \times \dfrac{x^2+3x-4}{x^2-6x+8}$

$= \dfrac{(x+2)(x-1)}{(x+3)(x+4)} \times \dfrac{(x-4)(x+3)}{(x-1)(x-1)} \times \dfrac{(x+4)(x-1)}{(x-2)(x-4)}$

$= \boldsymbol{\dfrac{x+2}{x-2}}$

(2) $\dfrac{a}{(c-a)(a-b)} + \dfrac{b}{(a-b)(b-c)} + \dfrac{c}{(b-c)(c-a)}$

$= \dfrac{a(b-c)+b(c-a)+c(a-b)}{(a-b)(b-c)(c-a)}$

$= \dfrac{ab-ac+bc-ba+ca-cb}{(a-b)(b-c)(c-a)}$

$= \boldsymbol{0}$

(3) $\left(\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}\right) \div \left(\dfrac{b}{a}+\dfrac{a}{b}\right)$

$= \left\{\dfrac{(a+b)^2+(a-b)^2}{(a-b)(a+b)}\right\} \div \left(\dfrac{b^2+a^2}{ab}\right)$

$= \left\{\dfrac{(a+b)^2+(a-b)^2}{a^2-b^2}\right\} \times \dfrac{ab}{b^2+a^2}$

$= \dfrac{ab(2a^2+2b^2)}{a^4-b^4} = \dfrac{2ab(a^2+b^2)}{(a^2+b^2)(a^2-b^2)}$

$= \boldsymbol{\dfrac{2ab}{a^2-b^2}}$

37 $\dfrac{1}{x(x-1)} + \dfrac{1}{(x-1)(x-2)} + \dfrac{1}{(x-2)(x-3)} + \dfrac{1}{(x-3)(x-4)}$

$= \left(\dfrac{1}{x-1}-\dfrac{1}{x}\right) + \left(\dfrac{1}{x-2}-\dfrac{1}{x-1}\right) + \left(\dfrac{1}{x-3}-\dfrac{1}{x-2}\right) + \left(\dfrac{1}{x-4}-\dfrac{1}{x-3}\right)$

$= -\dfrac{1}{x} + \dfrac{1}{x-4}$

$= \boldsymbol{\dfrac{4}{x(x-4)}}$

38 (1) $\dfrac{1}{x+1}+\dfrac{1}{x+3}-\dfrac{1}{x+5}-\dfrac{1}{x+7}$

$= \dfrac{(x+5)-(x+1)}{(x+1)(x+5)} + \dfrac{(x+7)-(x+3)}{(x+3)(x+7)}$

$= \dfrac{4}{(x+1)(x+5)} + \dfrac{4}{(x+3)(x+7)}$

$= \dfrac{4\{(x+3)(x+7)+(x+1)(x+5)\}}{(x+1)(x+3)(x+5)(x+7)}$

$= \dfrac{4(x^2+10x+21+x^2+6x+5)}{(x+1)(x+3)(x+5)(x+7)}$

$= \dfrac{4(2x^2+16x+26)}{(x+1)(x+3)(x+5)(x+7)}$

$= \boldsymbol{\dfrac{8(x^2+8x+13)}{(x+1)(x+3)(x+5)(x+7)}}$

(2) $\dfrac{x+1}{x}-\dfrac{x+7}{x+2}+\dfrac{x+8}{x+3}-\dfrac{x+6}{x+5}$

$$= \left(1+\frac{1}{x}\right)-\left(1+\frac{5}{x+2}\right)$$

$$+\left(1+\frac{5}{x+3}\right)-\left(1+\frac{1}{x+5}\right)$$

$$= \frac{1}{x}-\frac{5}{x+2}+\frac{5}{x+3}-\frac{1}{x+5}$$

$$= \frac{5}{x(x+5)}-\frac{5}{(x+2)(x+3)}$$

$$= \frac{5\{(x+2)(x+3)-x(x+5)\}}{x(x+2)(x+3)(x+5)}$$

$$= \frac{5(x^2+5x+6-x^2-5x)}{x(x+2)(x+3)(x+5)}$$

$$= \boldsymbol{\frac{30}{x(x+2)(x+3)(x+5)}}$$

39 (1) $\dfrac{1-\dfrac{1}{x-1}}{x+\dfrac{1}{1-\dfrac{1}{x}}}$

$$= \frac{1-\dfrac{1}{x-1}}{x+\dfrac{1}{1-\dfrac{1}{x}}\times\dfrac{x}{x}}$$

$$= \frac{1-\dfrac{1}{x-1}}{x+\dfrac{x}{x-1}}$$

$$= \frac{1-\dfrac{1}{x-1}}{x+\dfrac{x}{x-1}}\times\frac{(x-1)}{(x-1)}$$

$$= \frac{x-2}{x(x-1)+x}$$

$$= \boldsymbol{\frac{x-2}{x^2}}$$

(2) $\dfrac{x-\dfrac{1}{1+\dfrac{1}{x}}}{1-\dfrac{1}{x+1}}$

$$= \frac{x-\dfrac{1}{1+\dfrac{1}{x}}\times\dfrac{x}{x}}{1-\dfrac{1}{x+1}}$$

$$= \frac{x-\dfrac{x}{x+1}}{1-\dfrac{1}{x+1}}$$

$$= \frac{x-\dfrac{x}{x+1}}{1-\dfrac{1}{x+1}}\times\frac{x+1}{x+1}$$

$$= \frac{x(x+1)-x}{x+1-1}$$

$$= \frac{x^2}{x}$$

$$= \boldsymbol{x}$$

40 (1) $(2x^2-3x+5)\div(x-1)$

1	2	−3	+5
		2	−1
	2	−1	4

商 $\boldsymbol{2x-1}$，余り **4**

(2) $(3x^3-2x^2+x-1)\div(x-5)$

5	3	−2	+1	−1
		15	65	330
	3	13	66	329

商 $\boldsymbol{3x^2+13x+66}$，余り **329**

3節　数

41 (1) $\dfrac{1}{8}=\mathbf{0.125}$　(2) $\dfrac{3}{8}=\mathbf{0.375}$

(3) $\dfrac{10}{7}=\mathbf{1.\dot{4}2857\dot{1}}$

(4) $-\dfrac{5}{11}=\mathbf{-0.\dot{4}\dot{5}}$

42 (1) 2，$\dfrac{10}{2}$，$\sqrt{16}$，$(\sqrt{2})^2$

(2) 2，-4，0，$\dfrac{10}{2}$，$\sqrt{16}$，$(\sqrt{2})^2$

(3) 2，-4，0，$\dfrac{10}{2}$，$\dfrac{5}{4}$，$-\dfrac{9}{25}$，$\sqrt{16}$，$(\sqrt{2})^2$

(4) $-\sqrt{5}$，$\dfrac{2}{\sqrt{2}}$，π

43 (1) $|-7|=\mathbf{7}$

(2) $|2-\sqrt{5}|=-(2-\sqrt{5})=\boldsymbol{\sqrt{5}-2}$

(3) $|3-\pi|=-(3-\pi)=\boldsymbol{\pi-3}$

(4) $|2-\sqrt{3}|+|1-\sqrt{3}|$

$=(2-\sqrt{3})-(1-\sqrt{3})$

$=2-\sqrt{3}-1+\sqrt{3}=\mathbf{1}$

44 (1) $AB=|2-(-5)|=\mathbf{7}$

(2) $AB=|(-3)-(-1)|=\mathbf{2}$

45 (1) $\sqrt{16}=4$　(2) $\sqrt{121}=\mathbf{11}$

(3) $\sqrt{0.0016}=\mathbf{0.04}$

(4) $\sqrt{\dfrac{27}{16}}=\dfrac{\sqrt{27}}{\sqrt{16}}=\boldsymbol{\dfrac{3\sqrt{3}}{4}}$

(5) $(-\sqrt{7})^2=\mathbf{7}$

(6) $\sqrt{(-8)^2}=|-8|=\mathbf{8}$

(7) $-\sqrt{(-3)^2}=-|-3|=\mathbf{-3}$

(8) $\sqrt{(-18)(-2)}=\sqrt{36}=\mathbf{6}$

(9) $\sqrt{8}\sqrt{27}=2\sqrt{2}\cdot3\sqrt{3}=\boldsymbol{6\sqrt{6}}$

(10) $\sqrt{12}\sqrt{18}=2\sqrt{3}\cdot3\sqrt{2}=\boldsymbol{6\sqrt{6}}$

(11) $\dfrac{\sqrt{24}}{\sqrt{6}}=\dfrac{2\sqrt{6}}{\sqrt{6}}=\mathbf{2}$

(12) $\dfrac{\sqrt{75}}{\sqrt{3}}=\dfrac{5\sqrt{3}}{\sqrt{3}}=\mathbf{5}$

46 (1) $\sqrt{20}-\sqrt{45}=2\sqrt{5}-3\sqrt{5}$

$=\boldsymbol{-\sqrt{5}}$

(2) $\sqrt{28}+\sqrt{\dfrac{7}{4}}=2\sqrt{7}+\dfrac{\sqrt{7}}{2}=\boldsymbol{\dfrac{5\sqrt{7}}{2}}$

(3) $\sqrt{12}-4\sqrt{27}+3\sqrt{75}$

$=2\sqrt{3}-12\sqrt{3}+15\sqrt{3}=\boldsymbol{5\sqrt{3}}$

(4) $\sqrt{12}+\sqrt{27}-\sqrt{48}+\sqrt{75}-\sqrt{108}$

$=2\sqrt{3}+3\sqrt{3}-4\sqrt{3}+5\sqrt{3}-6\sqrt{3}$

$=\mathbf{0}$

(5) $(\sqrt{3}-\sqrt{6})^2$

$=(\sqrt{3})^2-2\sqrt{6}\sqrt{3}+(\sqrt{6})^2$

$=3-2\sqrt{18}+6=\boldsymbol{9-6\sqrt{2}}$

(6) $(2\sqrt{5}-\sqrt{2})(2\sqrt{5}+\sqrt{2})$

$=(2\sqrt{5})^2-(\sqrt{2})^2=20-2=\mathbf{18}$

(7) $\sqrt{7}(\sqrt{56}+\sqrt{21}-\sqrt{126})$

$=\sqrt{7}(2\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{3}-3\sqrt{7}\sqrt{2})$

$=7(2\sqrt{2}+\sqrt{3}-3\sqrt{2})=7(\sqrt{3}-\sqrt{2})$

$=\boldsymbol{7\sqrt{3}-7\sqrt{2}}$

(8) $(2\sqrt{5}+3\sqrt{2})(3\sqrt{5}-2\sqrt{2})$

$=30-4\sqrt{10}+9\sqrt{10}-12$

$=\boldsymbol{18+5\sqrt{10}}$

47 (1) $\dfrac{8}{3\sqrt{2}}=\dfrac{8\sqrt{2}}{3\cdot2}=\boldsymbol{\dfrac{4\sqrt{2}}{3}}$

(2) $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}=\dfrac{\sqrt{3}(\sqrt{3}-\sqrt{2})}{3}$

$=\boldsymbol{\dfrac{3-\sqrt{6}}{3}}$

(3) $\dfrac{1+\sqrt{7}}{2+\sqrt{7}}=\dfrac{(1+\sqrt{7})(2-\sqrt{7})}{(2+\sqrt{7})(2-\sqrt{7})}$

$=\dfrac{2+\sqrt{7}-7}{4-7}=\dfrac{-5+\sqrt{7}}{-3}=\boldsymbol{\dfrac{5-\sqrt{7}}{3}}$

(4) $\dfrac{2\sqrt{3}-3}{2\sqrt{3}+3}=\dfrac{(2\sqrt{3}-3)^2}{(2\sqrt{3}+3)(2\sqrt{3}-3)}$

$=\dfrac{12-12\sqrt{3}+9}{12-9}=\dfrac{21-12\sqrt{3}}{3}$

$=\boldsymbol{7-4\sqrt{3}}$

(5) $\dfrac{5+2\sqrt{3}}{4-\sqrt{3}}=\dfrac{(5+2\sqrt{3})(4+\sqrt{3})}{(4-\sqrt{3})(4+\sqrt{3})}$

$=\dfrac{20+13\sqrt{3}+6}{16-3}=\dfrac{26+13\sqrt{3}}{13}$

$=\boldsymbol{2+\sqrt{3}}$

(6) $\dfrac{1}{(\sqrt{2}-\sqrt{3})^2}=\dfrac{1}{5-2\sqrt{6}}$

$=\dfrac{5+2\sqrt{6}}{(5-2\sqrt{6})(5+2\sqrt{6})}$

$=\dfrac{5+2\sqrt{6}}{25-24}=\boldsymbol{5+2\sqrt{6}}$

48 (1) $x+y=(\sqrt{5}-\sqrt{2})+(\sqrt{5}+\sqrt{2})$

$=\boldsymbol{2\sqrt{5}}$

(2) $xy=(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

$=5-2=\mathbf{3}$

(3) $(x-2)(y-2)=xy-2x-2y+4$

$=xy-2(x+y)+4=3-2\cdot2\sqrt{5}+4$

$=\boldsymbol{7-4\sqrt{5}}$

(4) x^2+y^2

$=(x+y)^2-2xy$

$=(2\sqrt{5})^2-2\cdot3=20-6=\mathbf{14}$

(5) $x^3+y^3=(x+y)(x^2-xy+y^2)$

$=2\sqrt{5}(14-3)=2\sqrt{5}\cdot11=\boldsymbol{22\sqrt{5}}$

(6) $x^3y+xy^3=xy(x^2+y^2)=3\cdot14=\mathbf{42}$

49 (1) **$x=-4$ のとき**

$|-4+2|=|-2|=\mathbf{2}$

$x=-2$ のとき

$|-2+2|=|0|=\mathbf{0}$

$x=0$ のとき

$|0+2|=|2|=\mathbf{2}$

$x=5$ のとき

$|5+2|=|7|=\mathbf{7}$

(2) **$x=-4$ のとき**

$|5-(-4)|=|5+4|=|9|=\mathbf{9}$

$\boldsymbol{x=-2}$ **のとき**

$|5-(-2)|=|5+2|=|7|=\mathbf{7}$

$\boldsymbol{x=0}$ **のとき**

$|5-0|=|5|=\mathbf{5}$

$\boldsymbol{x=5}$ **のとき**

$|5-5|=|0|=\mathbf{0}$

(3)　$\boldsymbol{x=-4}$ **のとき**

$|-4+3|+|-4-2|$

$=|-1|+|-6|=1+6=\mathbf{7}$

$\boldsymbol{x=-2}$ **のとき**

$|-2+3|+|-2-2|=|1|+|-4|$

$=1+4=\mathbf{5}$

$\boldsymbol{x=0}$ **のとき**

$|0+3|+|0-2|=|3|+|-2|$

$=3+2=\mathbf{5}$

$\boldsymbol{x=5}$ **のとき**

$|5+3|+|5-2|=|8|+|3|=8+3$

$=\mathbf{11}$

50　(1)　$\sqrt{x^2-6x+9}=\sqrt{(x-3)^2}$

$=|x-3|$

$\boldsymbol{x \geqq 3}$ **のとき**　$\boldsymbol{x-3}$

$\boldsymbol{x<3}$ **のとき**　$\boldsymbol{-x+3}$

(2)　$\sqrt{x^2}+\sqrt{(2x-5)^2}=|x|+|2x-5|$

$\boldsymbol{x \geqq \frac{5}{2}}$ **のとき**　$x+2x-5=\boldsymbol{3x-5}$

$\boldsymbol{0 \leqq x < \frac{5}{2}}$ **のとき**

$x-(2x-5)=x-2x+5=\boldsymbol{-x+5}$

$\boldsymbol{x<0}$ **のとき**

$-x-(2x-5)=-x-2x+5$

$=\boldsymbol{-3x+5}$

51　$P=\sqrt{x+1}+\sqrt{x+4a+1}$

$=\sqrt{a^2-2a+1}+\sqrt{a^2-2a+4a+1}$

$=\sqrt{(a-1)^2}+\sqrt{(a+1)^2}$

$=|a-1|+|a+1|$

$-1<a<1$ より

$P=-(a-1)+(a+1)$

$=-a+1+a+1=\mathbf{2}$

52　(1)　$x^2+\dfrac{1}{x^2}=\left(x+\dfrac{1}{x}\right)^2-2$

$=3^2-2=\mathbf{7}$

(2)　$x^3+\dfrac{1}{x^3}=\left(x+\dfrac{1}{x}\right)\left(x^2-1+\dfrac{1}{x^2}\right)$

$=3(7-1)=\mathbf{18}$

(3)　$\left(x-\dfrac{1}{x}\right)^2=x^2+\dfrac{1}{x^2}-2=7-2=5$

より

$x-\dfrac{1}{x}=\boldsymbol{\pm\sqrt{5}}$

53　$\dfrac{\sqrt{2}+1}{\sqrt{2}-1}=\dfrac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$=\dfrac{2+2\sqrt{2}+1}{2-1}=\boldsymbol{3+2\sqrt{2}}$

$\sqrt{2}=1.4142\cdots$ であるから

$3+2\sqrt{2}=5.8282\cdots$

(1)　$a=\mathbf{5}$

(2)　$b=(3+2\sqrt{2})-a=3+2\sqrt{2}-5$

$=\boldsymbol{2\sqrt{2}-2}$

(3)　$ab+b^2-b=b(a+b-1)$

$=(2\sqrt{2}-2)(5+2\sqrt{2}-2-1)$

$=(2\sqrt{2}-2)(2\sqrt{2}+2)=8-4=\mathbf{4}$

54　(1)　$\dfrac{1}{1+\sqrt{2}+\sqrt{3}}$

$=\dfrac{1+\sqrt{2}-\sqrt{3}}{\{(1+\sqrt{2})+\sqrt{3}\}\{(1+\sqrt{2})-\sqrt{3}\}}$

$=\dfrac{1+\sqrt{2}-\sqrt{3}}{(1+\sqrt{2})^2-(\sqrt{3})^2}=\dfrac{1+\sqrt{2}-\sqrt{3}}{2\sqrt{2}}$

$=\boldsymbol{\dfrac{2+\sqrt{2}-\sqrt{6}}{4}}$

(2)　$\dfrac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}+\dfrac{\sqrt{2}-\sqrt{3}+\sqrt{5}}{\sqrt{2}-\sqrt{3}-\sqrt{5}}$

$=\dfrac{(\sqrt{2}+\sqrt{3}+\sqrt{5})^2}{(\sqrt{2}+\sqrt{3}-\sqrt{5})(\sqrt{2}+\sqrt{3}+\sqrt{5})}$

$+\dfrac{(\sqrt{2}-\sqrt{3}+\sqrt{5})^2}{(\sqrt{2}-\sqrt{3}-\sqrt{5})(\sqrt{2}-\sqrt{3}+\sqrt{5})}$

$=\dfrac{\{(\sqrt{2}+\sqrt{5})+\sqrt{3}\}^2}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2}$

$+\dfrac{\{(\sqrt{2}+\sqrt{5})-\sqrt{3}\}^2}{(\sqrt{2}-\sqrt{3})^2-(\sqrt{5})^2}$

$=\dfrac{(\sqrt{2}+\sqrt{5})^2+2\sqrt{3}(\sqrt{2}+\sqrt{5})+(\sqrt{3})^2}{2\sqrt{6}}$

$+\dfrac{(\sqrt{2}+\sqrt{5})^2-2\sqrt{3}(\sqrt{2}+\sqrt{5})+(\sqrt{3})^2}{-2\sqrt{6}}$

$=\dfrac{4\sqrt{3}(\sqrt{2}+\sqrt{5})}{2\sqrt{6}}=\sqrt{2}(\sqrt{2}+\sqrt{5})$

$= \mathbf{2+\sqrt{10}}$

55 (左辺) $= \sqrt{(a+b)-2\sqrt{ab}}$

$= \sqrt{(\sqrt{a})^2 - 2\sqrt{a}\sqrt{b} + (\sqrt{b})^2}$

$= \sqrt{(\sqrt{a}-\sqrt{b})^2}$

$= \sqrt{a} - \sqrt{b} =$ (右辺)

56 (1) $\sqrt{5+2\sqrt{6}} = \sqrt{(3+2)+2\sqrt{3\cdot 2}}$

$= \mathbf{\sqrt{3}+\sqrt{2}}$

(2) $\sqrt{8-2\sqrt{12}} = \sqrt{(6+2)-2\sqrt{6\cdot 2}}$

$= \mathbf{\sqrt{6}-\sqrt{2}}$

(3) $\sqrt{15+2\sqrt{54}} = \sqrt{(9+6)+2\sqrt{9\cdot 6}}$

$= \sqrt{9}+\sqrt{6} = \mathbf{3+\sqrt{6}}$

(4) $\sqrt{9+\sqrt{80}} = \sqrt{9+2\sqrt{20}}$

$= \sqrt{(5+4)+2\sqrt{5\cdot 4}} = \sqrt{5}+\sqrt{4}$

$= \mathbf{\sqrt{5}+2}$

(5) $\sqrt{9-\sqrt{56}} = \sqrt{9-2\sqrt{14}}$

$= \sqrt{(7+2)-2\sqrt{7\cdot 2}} = \mathbf{\sqrt{7}-\sqrt{2}}$

(6) $\sqrt{4+\sqrt{15}} = \sqrt{\dfrac{8+2\sqrt{15}}{2}}$

$= \dfrac{\sqrt{(5+3)+2\sqrt{5\cdot 3}}}{\sqrt{2}} = \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{2}}$

$= \mathbf{\dfrac{\sqrt{10}+\sqrt{6}}{2}}$

(7) $\sqrt{11-\sqrt{96}} = \sqrt{11-2\sqrt{24}}$

$= \sqrt{(8+3)-2\sqrt{8\cdot 3}}$

$= \sqrt{8}-\sqrt{3} = \mathbf{2\sqrt{2}-\sqrt{3}}$

(8) $\sqrt{3+\sqrt{5}} = \sqrt{\dfrac{6+2\sqrt{5}}{2}}$

$= \dfrac{\sqrt{(5+1)+2\sqrt{5\cdot 1}}}{\sqrt{2}}$

$= \dfrac{\sqrt{5}+1}{\sqrt{2}} = \mathbf{\dfrac{\sqrt{10}+\sqrt{2}}{2}}$

(9) $\sqrt{8+3\sqrt{7}} = \sqrt{8+\sqrt{63}}$

$= \sqrt{\dfrac{16+2\sqrt{63}}{2}} = \dfrac{\sqrt{(9+7)+2\sqrt{9\cdot 7}}}{\sqrt{2}}$

$= \dfrac{\sqrt{9}+\sqrt{7}}{\sqrt{2}} = \dfrac{3+\sqrt{7}}{\sqrt{2}} = \mathbf{\dfrac{3\sqrt{2}+\sqrt{14}}{2}}$

57 (1) $\sqrt{1+x-2\sqrt{x}}$

$= \sqrt{(x+1)-2\sqrt{x\cdot 1}} = \mathbf{\sqrt{x}-1}$

(2) $\sqrt{1+2\sqrt{a(1-a)}}$

$= \sqrt{\{a+(1-a)\}+2\sqrt{a(1-a)}}$

$= \mathbf{\sqrt{a}+\sqrt{1-a}}$

1章の問題　A

1 (1) $(ab^2)^3 \times (a^2b)^2 = a^3b^6 \times a^4b^2 = \mathbf{a^7b^8}$

(2) $\dfrac{(a^3b^2)^3}{(a^2b)^2} = \dfrac{a^9b^6}{a^4b^2} = \mathbf{a^5b^4}$

(3) $\dfrac{(a^3b^2)^3\cdot a^2}{a^4b^2} = \dfrac{a^{11}b^6}{a^4b^2} = \mathbf{a^7b^4}$

(4) $\sqrt[3]{\sqrt{a}} = (a^{\frac{1}{2}})^{\frac{1}{3}} = \mathbf{a^{\frac{1}{6}}}$

2 (1) $6x^2-x-15 = \mathbf{(2x+3)(3x-5)}$

(2) $8x^3-36x^2y+54xy^2-27y^3$

$= (2x)^3 - 3\cdot(2x)^2\cdot 3y + 3\cdot 2x\cdot(3y)^2$

$- (3y)^3$

$= \mathbf{(2x-3y)^3}$

3 (1) $\dfrac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}$

$= \dfrac{(\sqrt{6}+\sqrt{2})^2}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})}$

$= \dfrac{6+2\sqrt{12}+2}{6-2} = \dfrac{8+4\sqrt{3}}{4}$

$= \mathbf{2+\sqrt{3}}$

(2) $\dfrac{3\sqrt{2}-1}{\sqrt{2}-1} = \dfrac{(3\sqrt{2}-1)(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$= \dfrac{6+3\sqrt{2}-\sqrt{2}-1}{2-1} = \mathbf{5+2\sqrt{2}}$

4 $(x-1)(x+2)(x+b)$

$= (x^2+x-2)(x+b)$

$= x^3+(b+1)x^2+(b-2)x-2b$

したがって

x^3+4x^2+x-a

$= x^3+(b+1)x^2+(b-2)x-2b$

より　$4 = b+1$

$1 = b-2$

$-a = -2b$　なので　$\mathbf{a=6}$　$\mathbf{b=3}$

5 最大公約数が $x+1$ なので，A も B も $(x+1)$ を因数にもつ

$x^4-1 = (x^2+1)(x^2-1)$

$= (x^2+1)(x+1)(x-1)$，A は 2 次式，B は 3 次式より

$\mathbf{A} = (x+1)(x-1) = \mathbf{x^2-1}$

$\mathbf{B} = (x^2+1)(x+1) = \mathbf{x^3+x^2+x+1}$

6 $x+y = \sqrt{3}+1+\sqrt{3}-1 = 2\sqrt{3}$　より

$$x^2+xy=x(x+y)$$
$$=(\sqrt{3}+1)\cdot 2\sqrt{3}$$
$$=\mathbf{6+2\sqrt{3}}$$

7 (1)　$a^2-b^2=(a+b)(a-b)$

$$=\left(\frac{1}{\sqrt{7}-\sqrt{3}}+\frac{1}{\sqrt{7}+\sqrt{3}}\right)\times\left(\frac{1}{\sqrt{7}-\sqrt{3}}-\frac{1}{\sqrt{7}+\sqrt{3}}\right)$$
$$=\left(\frac{(\sqrt{7}+\sqrt{3})+(\sqrt{7}-\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})}\right)\times\left(\frac{(\sqrt{7}+\sqrt{3})-(\sqrt{7}-\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})}\right)$$
$$=\frac{2\sqrt{7}}{4}\cdot\frac{2\sqrt{3}}{4}$$
$$=\mathbf{\frac{\sqrt{21}}{4}}$$

別解　$a=\dfrac{\sqrt{7}+\sqrt{3}}{4}$，$b=\dfrac{\sqrt{7}-\sqrt{3}}{4}$ と変形して

$$a^2-b^2=(a+b)(a-b)$$
$$=\left(\frac{\sqrt{7}+\sqrt{3}}{4}+\frac{\sqrt{7}-\sqrt{3}}{4}\right)\times\left(\frac{\sqrt{7}+\sqrt{3}}{4}-\frac{\sqrt{7}-\sqrt{3}}{4}\right)$$
$$=\frac{2\sqrt{7}}{4}\cdot\frac{2\sqrt{3}}{4}$$
$$=\frac{\sqrt{7}}{2}\cdot\frac{\sqrt{3}}{2}=\mathbf{\frac{\sqrt{21}}{4}}$$

(2)　$a^3+b^3=(a+b)(a^2-ab+b^2)$

$$=(a+b)\{(a+b)^2-3ab\}$$
$$=\left(\frac{1}{\sqrt{7}-\sqrt{3}}+\frac{1}{\sqrt{7}+\sqrt{3}}\right)\times\left\{\left(\frac{1}{\sqrt{7}-\sqrt{3}}+\frac{1}{\sqrt{7}+\sqrt{3}}\right)^2-3\cdot\frac{1}{\sqrt{7}-\sqrt{3}}\cdot\frac{1}{\sqrt{7}+\sqrt{3}}\right\}$$
$$=\frac{2\sqrt{7}}{4}\left\{\left(\frac{2\sqrt{7}}{4}\right)^2-\frac{3}{4}\right\}$$
$$=\frac{\sqrt{7}}{2}\left(\frac{7}{4}-\frac{3}{4}\right)$$
$$=\frac{\sqrt{7}}{2}\left(\frac{4}{4}\right)$$
$$=\mathbf{\frac{\sqrt{7}}{2}}$$

別解　$a^3+b^3=(a+b)^3-3ab(a+b)$

$$a+b=\frac{\sqrt{7}}{2}$$
$$a\cdot b=\frac{4}{16}=\frac{1}{4}$$
$$=\left(\frac{\sqrt{7}}{2}\right)^3-\frac{3}{4}\left(\frac{\sqrt{7}}{2}\right)$$
$$=\frac{7\sqrt{7}}{8}-\frac{3\sqrt{7}}{8}=\frac{4\sqrt{7}}{8}=\mathbf{\frac{\sqrt{7}}{2}}$$

8 (1)　$x+y$

$$=\sqrt{5}+\sqrt{3}-\sqrt{2}+\sqrt{5}-\sqrt{3}+\sqrt{2}$$
$$=\mathbf{2\sqrt{5}}$$

(2)　xy

$$=(\sqrt{5}+\sqrt{3}-\sqrt{2})(\sqrt{5}-\sqrt{3}+\sqrt{2})$$
$$=\{\sqrt{5}+(\sqrt{3}-\sqrt{2})\}\times\{\sqrt{5}-(\sqrt{3}-\sqrt{2})\}$$

$\sqrt{3}-\sqrt{2}=A$ とおくと

$$=(\sqrt{5}+A)(\sqrt{5}-A)$$
$$=5-A^2=5-(\sqrt{3}-\sqrt{2})^2$$
$$=5-(3-2\sqrt{6}+2)$$
$$=\mathbf{2\sqrt{6}}$$

(3)　$\dfrac{1}{x^2}+\dfrac{1}{y^2}=\dfrac{x^2+y^2}{x^2y^2}$

$$=\frac{(x+y)^2-2xy}{(xy)^2}$$
$$=\frac{(2\sqrt{5})^2-2\cdot 2\sqrt{6}}{(2\sqrt{6})^2}$$
$$=\frac{20-4\sqrt{6}}{24}=\mathbf{\frac{5-\sqrt{6}}{6}}$$

9　$\dfrac{x-\dfrac{1}{x}}{1-\dfrac{1}{x}}=\dfrac{x-\dfrac{1}{x}}{1-\dfrac{1}{x}}\cdot\dfrac{x}{x}$

$$=\frac{x^2-1}{x-1}$$
$$=\frac{(x+1)(x-1)}{x-1}$$
$$=\mathbf{x+1}$$

1章の問題　B

1 (1)　$x^2-2x=A$ とおくと

$$(x^2-2x-16)(x^2-2x-14)+1$$
$$=(A-16)(A-14)+1$$
$$=A^2-30A+225$$

$= (A-15)^2$

$= (x^2-2x-15)^2$

$= \boldsymbol{(x-5)^2(x+3)^2}$

(2) $8x^3+12x^2y+4xy^2+6x^2+9xy+3y^2$

$= 4x(2x^2+3xy+y^2)$
$\qquad +3(2x^2+3xy+y^2)$

$= 4x(2x+y)(x+y)+3(2x+y)(x+y)$

$= \boldsymbol{(4x+3)(2x+y)(x+y)}$

2 (1) $(x^2+xy+y^2)(x^2-xy+y^2)$
$\qquad (x^4-x^2y^2+y^4)$

$= (x^2+y^2+xy)(x^2+y^2-xy)$
$\qquad (x^4-x^2y^2+y^4)$

$x^2+y^2=A$ とおくと

$= (A+xy)(A-xy)(x^4-x^2y^2+y^4)$

$= (A^2-x^2y^2)(x^4-x^2y^2+y^4)$

$= (x^4+y^4+x^2y^2)(x^4+y^4-x^2y^2)$

$x^4+y^4=B$ とおくと

$= (B+x^2y^2)(B-x^2y^2)$

$= B^2-x^4y^4$

$= \boldsymbol{x^8+x^4y^4+y^8}$

(2) $(x-b)(x-c)(b-c)$
$\qquad +(x-c)(x-a)(c-a)$
$\qquad +(x-a)(x-b)(a-b)$

$= (x^2-bx-cx+bc)(b-c)$
$\qquad +(x^2-ax-cx+ac)(c-a)$
$\qquad +(x^2-ax-bx+ab)(a-b)$

$= bx^2-b^2x-bcx+b^2c$
$\qquad -cx^2+cbx+c^2x-bc^2$
$\qquad +cx^2-acx-c^2x+ac^2$
$\qquad -ax^2+a^2x+acx-a^2c$
$\qquad +ax^2-a^2x-abx+a^2b$
$\qquad -bx^2+abx+b^2x-ab^2$

$= \boldsymbol{a^2b-a^2c-ab^2+b^2c+ac^2-bc^2}$

(3) $bc(b+c)+ca(c+a)$
$\qquad +ab(a+b)+2abc$

$= b^2c+bc^2+c^2a+ca^2+a^2b+ab^2$
$\qquad +2abc$

$= (b+c)a^2+(b^2+2bc+c^2)a$
$\qquad +bc(b+c)$

$= (b+c)a^2+(b+c)^2a+bc(b+c)$

$= (b+c)\{a^2+(b+c)a+bc\}$

$= \boldsymbol{(a+b)(b+c)(c+a)}$

(4) $(a+b+c)^3-a^3-b^3-c^3$

$= \{(a+b+c)^3-a^3\}-(b^3+c^3)$

$= (a+b+c-a)$
$\qquad \{(a+b+c)^2+(a+b+c)a+a^2\}$
$\qquad -(b+c)(b^2-bc+c^2)$

$= (b+c)(a^2+b^2+c^2+2ab+2bc+2ac$
$\qquad +a^2+ab+ac+a^2-b^2+bc-c^2)$

$= 3(b+c)\{a^2+(b+c)a+bc\}$

$= \boldsymbol{3(a+b)(b+c)(c+a)}$

3 (1) $\dfrac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$
$\qquad +\dfrac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$

$= \dfrac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{(\sqrt{2}+\sqrt{3}-\sqrt{5})(\sqrt{2}+\sqrt{3}+\sqrt{5})}$
$\quad +\dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{(\sqrt{2}+\sqrt{3}+\sqrt{5})(\sqrt{2}+\sqrt{3}-\sqrt{5})}$

$= \dfrac{2\sqrt{2}+2\sqrt{3}}{(\sqrt{2}+\sqrt{3})^2-5}$

$= \dfrac{2\sqrt{2}+2\sqrt{3}}{5+2\sqrt{6}-5}$

$= \dfrac{2\sqrt{2}+2\sqrt{3}}{2\sqrt{6}}$

$= \dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{6}} = \boldsymbol{\dfrac{2\sqrt{3}+3\sqrt{2}}{6}}$

(2) $(4+\sqrt{2}+\sqrt{3})(4-\sqrt{2}+\sqrt{3})$
$\qquad (4+\sqrt{2}-\sqrt{3})(4-\sqrt{2}-\sqrt{3})$

$= \{4+(\sqrt{2}+\sqrt{3})\}\{4-(\sqrt{2}-\sqrt{3})\}$
$\qquad \{4+(\sqrt{2}-\sqrt{3})\}\{4-(\sqrt{2}+\sqrt{3})\}$

$= \{4^2-(\sqrt{2}+\sqrt{3})^2\}$
$\qquad \times\{4^2-(\sqrt{2}-\sqrt{3})^2\}$

$= (16-5-2\sqrt{6})(16-5+2\sqrt{6})$

$= (11-2\sqrt{6})(11+2\sqrt{6})$

$= 121-24 = \boldsymbol{97}$

(3) $\sqrt{3+\sqrt{5}}+\sqrt{3-\sqrt{5}}$

$= \dfrac{\sqrt{6+2\sqrt{5}}}{\sqrt{2}}+\dfrac{\sqrt{6-2\sqrt{5}}}{\sqrt{2}}$

$= \dfrac{\sqrt{1+5+2\sqrt{1\cdot 5}}}{\sqrt{2}}+\dfrac{\sqrt{1+5-2\sqrt{1\cdot 5}}}{\sqrt{2}}$

$= \dfrac{1+\sqrt{5}}{\sqrt{2}}+\dfrac{\sqrt{5}-1}{\sqrt{2}} = \dfrac{2\sqrt{5}}{\sqrt{2}}$

$= \boldsymbol{\sqrt{10}}$

4　(1)　$(x+y)^2=x^2+2xy+y^2$　より

$4^2=2xy+20$

$-2xy=20-16$

$-2xy=4$

したがって

$xy=\mathbf{-2}$

(2)　$(x-y)^2=x^2-2xy+y^2$

$=20-2\cdot(-2)=20+4=24$

したがって　$x-y=\pm\sqrt{24}=\pm2\sqrt{6}$

$x>y$ より　$x-y=\mathbf{2\sqrt{6}}$

(3)　$x^3-y^3=(x-y)(x^2+xy+y^2)$

$=2\sqrt{6}\,(20-2)$

$=\mathbf{36\sqrt{6}}$

5　(1)

$$\begin{array}{r|l} & x^2+(a-1)x+\{-2-(a-1)\} \\ \hline x+1 & x^3+ax^2-2x-1 \\ & \underline{x^3+x^2} \\ & (a-1)x^2-2x-1 \\ & \underline{(a-1)x^2+(a-1)x} \\ & \{-2-(a-1)\}x-1 \\ & \underline{\{-2-(a-1)\}x+\{-2-(a-1)\}} \\ & -1-\{-2-(a-1)\} \end{array}$$

割り切れるので，余りは0　したがって

$-1-\{-2-(a-1)\}=0$

$-1+2+a-1=0$

$\boldsymbol{a=0}$

(2)　$6x^3+4x^2+x+2$

$=(ax-1)(2x^2+2x+b)+3$

$=2ax^3+2ax^2+abx-2x^2-2x-b+3$

$=2ax^3+(2a-2)x^2+(ab-2)x-b+3$

両辺の係数を比較して

$6=2a$

$4=2a-2$

$1=ab-2$

$2=-b+3$

これを解いて　$\boldsymbol{a=3,\ b=1}$

6　$\sqrt{14+6\sqrt{5}}=\sqrt{14+2\sqrt{45}}$

$=\sqrt{9+5+2\sqrt{9\cdot5}}=\sqrt{9}+\sqrt{5}=3+\sqrt{5}$

(1)　$\sqrt{5}=2.236\cdots$ であるから

$3+\sqrt{5}=5.236\cdots$

したがって　$a=\mathbf{5}$

(2)　$b=3+\sqrt{5}-5=\sqrt{5}-2$　である。

$b^2+\dfrac{1}{b^2}=\left(b+\dfrac{1}{b}\right)^2-2$

$=\left(\sqrt{5}-2+\dfrac{1}{\sqrt{5}-2}\right)^2-2$

$=\left(\sqrt{5}-2+\dfrac{\sqrt{5}+2}{(\sqrt{5}-2)(\sqrt{5}+2)}\right)^2-2$

$=(\sqrt{5}-2+\sqrt{5}+2)^2-2$

$=(2\sqrt{5})^2-2$

$=20-2=\mathbf{18}$

(3)　$b^3+\dfrac{1}{b^3}=\left(b+\dfrac{1}{b}\right)\left(b^2-1+\dfrac{1}{b^2}\right)$

$=(2\sqrt{5})(18-1)$

$=\mathbf{34\sqrt{5}}$

7　$1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{x}}}}}$

$=1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{x}}\times\cfrac{x}{x}}}}$

$=1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{x}{x-1}}\times\cfrac{x-1}{x-1}}}$

$=1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{x-1}{x-1-x}}}$

$=1-\cfrac{1}{1-\cfrac{1}{1-\cfrac{x-1}{-1}}}=1-\cfrac{1}{1-\cfrac{1}{x}}\times\cfrac{x}{x}$

$=1-\dfrac{x}{x-1}=\dfrac{x-1-x}{x-1}=\dfrac{-1}{x-1}$

$=\boldsymbol{-\dfrac{1}{x-1}}$